Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 01-m

Advanced Control Methods

Contents

Reference:

"Modern Control Engineering" Ogata

"State Space and Linear Systems" Schaum's Outline Series

" Feedback Control Systems" Schaum's Outline Series

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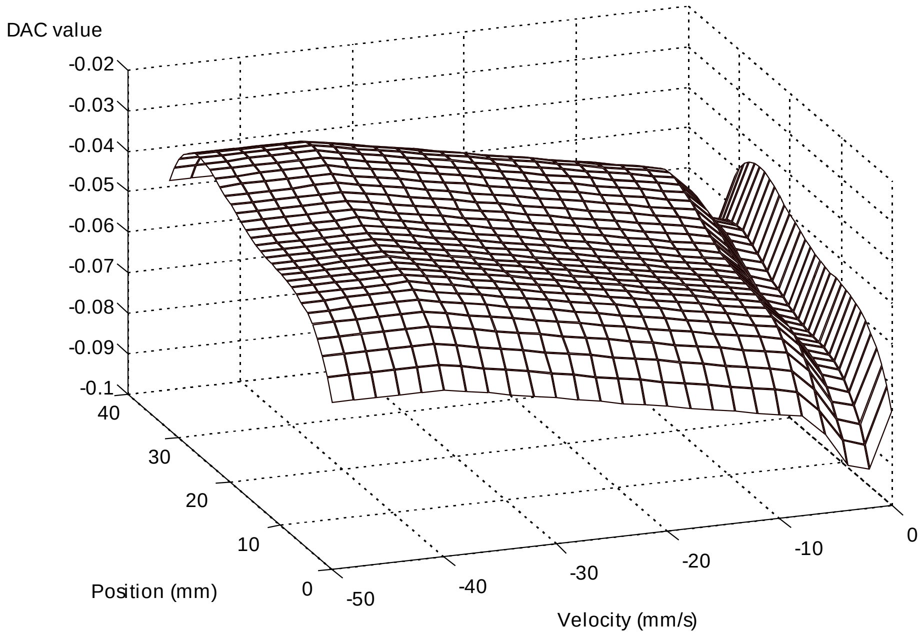
1. Robust Control

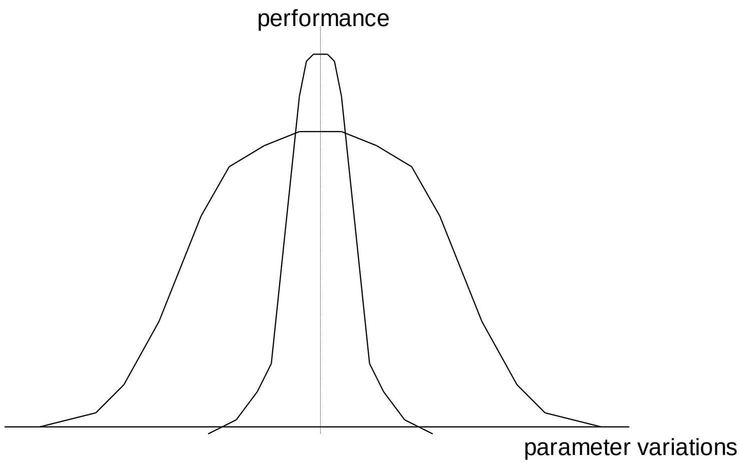
Robust System: A system that maintains its original intended purpose despite system parameter variations.

Examples of parameter variations:

1. Change of mechanical geometry (e.g. in robot manipulators)
2. Change of electrical parameters (e.g. motor gets hot)
3. Mechanical uncertainties (e.g. friction, misalignment)
4. Change of load (e.g. in pick and place machines)
5. Other factors (e.g. sensor resolution, noise, vibration)

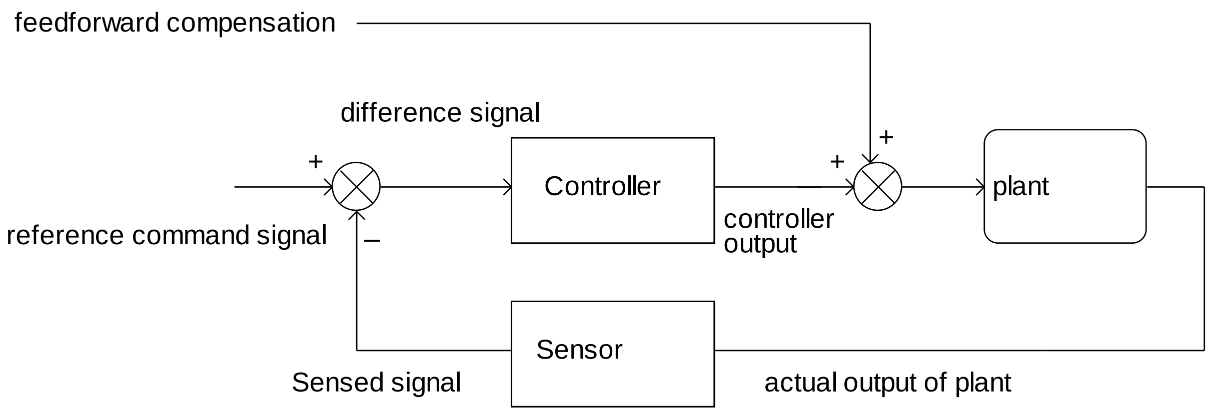
An example of friction model (a linear slide):





2. Feedforward Compensation

- A way to predict performance change – Predictive Systems
- Compensate the change without waiting for feedback sensor signals.
- Useful for repetitive motions
- Useful for known load change



How to generate the feed forward compensation signal? Examples:

High speed motion: acceleration compensation	Position command > d/dt > velocity > d/dt > acceleration > x K > to f/f
Spring loaded system: position compensation	Position command > x K > to f/f
Fluid stirring: Velocity compensation	Position > d/dt > velocity > x K > to f/f
Multi-axis system: Motion decoupling	Design according to the structure of the motion mechanics.

3. Controllability and Observability

The concept of *controllability* addresses the question of whether it is possible to *control* or *steer* the state (vector) **x** from the input **u**. Specifically, does there exist a physically realizable input **u** that can be applied to the plant over a finite period of time that will steer the entire state vector **x** (every one of the n components of **x**) from any point $\mathbf{x}_0$ in *state space* to any other point $\mathbf{x}_1$? If yes, the plant is **controllable**; if no, it is **uncontrollable**.

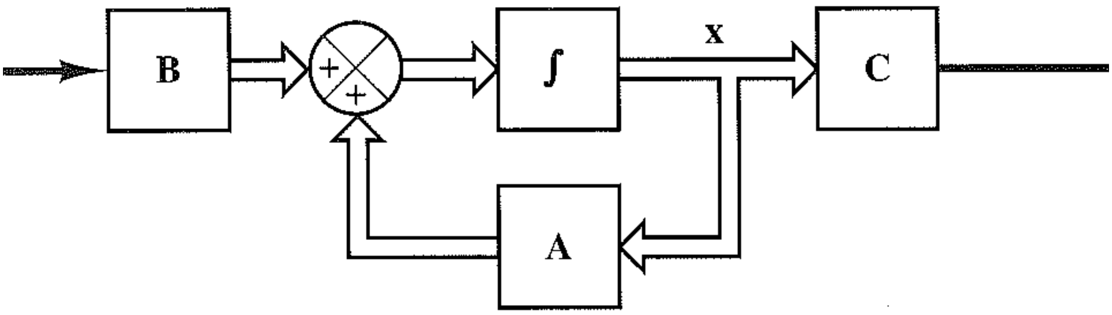
The concept of *observability* is complementary to that of controllability. It addresses the question of whether it is possible to determine all of the n components of the state vector **x** by measurement of the output **y** over a finite period of time. If yes, the system is **observable**; if no, it is **unobservable**. Obviously, if $\mathbf{y} = \mathbf{x}$, that is, if all state variables are measured, the system is observable. However, if $\mathbf{y} \neq \mathbf{x}$ and C is not a square matrix, the plant may still be observable.

Linear, time-invariant plant models in state variable form [Equations (*3.25b*) or (*3.36*)] are **controllable** if and only if the following **controllability matrix** has rank n (n linearly independent columns), where n is the number of state variables in the state vector **x**:

$$\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} \qquad (20.1)$$

Similarly, the plant model is **observable** if and only if the following **observability matrix** has rank n (n linearly independent rows):

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad (20.2)$$



Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

4. State Feedback and Observer

If the plant is *controllable*, the matrix G exists that can yield any (arbitrary) set of desired roots for the characteristic equation of this closed-loop system, represented by $|\lambda I - A + BG| = 0$, where the λ solutions of this determinant equation are the roots. This is the basic result.

EXAMPLE 20.2. A block diagram of the state feedback system given by Equations (*20.3*) and (*20.4*) is shown in Fig. 20-1.

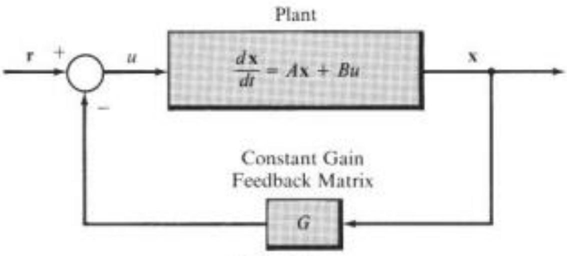


Fig. 20-1

To implement a state feedback design, the entire state vector $\mathbf{x}$ must somehow be made available, either as $\mathbf{x}$ exactly, or as an adequate approximation, denoted $\hat{\mathbf{x}}$. If the output is $\mathbf{y} = \mathbf{x}$, as in Fig. 20-1, there obviously is no problem. But, if all states are not available as outputs, which is more common, then *observability* of the plant model differential and output equations ($d\mathbf{x}/dt = A\mathbf{x} + Bu$ and $\mathbf{y} = C\mathbf{x}$) is required to obtain the needed state *estimate* or *observer* $\hat{\mathbf{x}}$. The equations for a typical state observer system are given by

$$\frac{d\hat{\mathbf{x}}}{dt} = (A - LC)\hat{\mathbf{x}} + L\mathbf{y} + B\mathbf{u} \qquad (20.5)$$

where A, B, and C are matrices of the plant and output measurement systems and L is an *observer design matrix* to be determined in a particular problem.

EXAMPLE 20.3. A detailed block diagram of the state observer system given by Equation (*20.5*) is shown in Fig. 20-2, along with the plant and measurement system block diagram (upper portion) for generating the needed input signals for the observer system (lower portion).

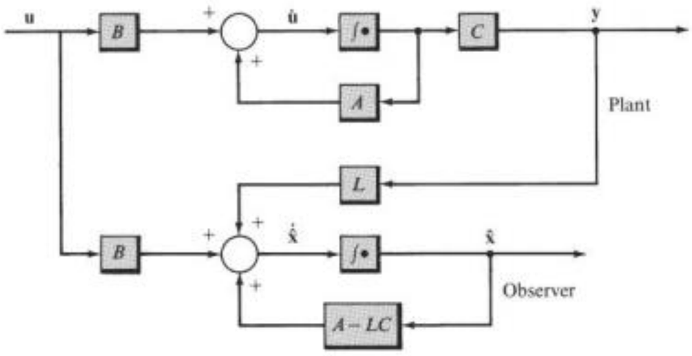


Fig. 20-2

EXAMPLE 20.4. Under suitable conditions, which include controllability and observability of the plant to be controlled, a *separation principle* applies and the state feedback portion (matrix G) and observer portion (matrix L) of a state feedback control system (with $\mathbf{y} \neq \mathbf{x}$) can be designed independently. A block diagram of the combined systems is shown in Fig. 20-3.

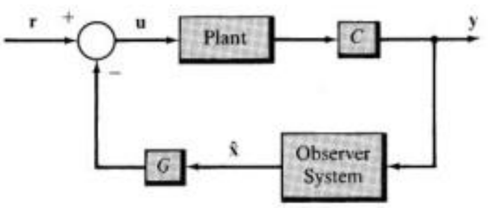


Fig. 20-3

We have omitted many details in this introductory material, and state feedback control systems are often more complex than described above.

5. Optimal Control

The design problems discussed in earlier chapters are, in an elementary sense, optimal control problems. The classical measures of system performance such as steady state error, gain margin, and phase margin are essentially criteria of optimality, and control system compensators are designed to meet these requirements. In more general optimal control problems, the system measure of performance, or *performance index*, is not fixed beforehand. Instead, compensation is chosen so that the performance index is *maximized* or *minimized*. The value of the performance index is unknown until the completion of the optimization process.

In many problems, the performance index is a measure or function of the error $e(t)$ between the actual and ideal responses. It is formulated in terms of the design parameters chosen, to optimize the performance index, subject to existing physical constraints.

EXAMPLE 20.7. For the system illustrated in Fig. 20-5 we want to find a $K \geq 0$ such that the integral of the square of the error e is minimized when the input is a unit step function. Since $e \equiv e(t)$ is not constant, but a function of time, we can formulate this problem as follows: Choose $K \geq 0$ such that $\int_0^\infty e^2(t)\,dt$ is minimized, where

$$e(t) = \mathscr{L}^{-1}\left[\frac{s+2}{s^2+2s+K}\right] = \sqrt{\frac{K}{K-1}}\, e^{-t}\sin\left(\sqrt{K-1}\,t + \tan^{-1}\sqrt{K-1}\right)$$

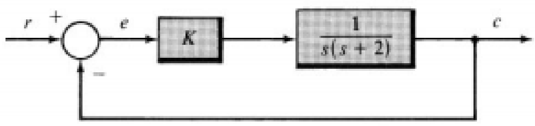


Fig. 20-5

The solution may be obtained for $K > 1$ using conventional minimization techniques of integral calculus, as follows:

$$\int_0^\infty e^2(t)\,dt = \frac{K}{K-1}\int_0^\infty \left[e^{-t}\sin\left(\sqrt{K-1}\,t + \tan^{-1}\sqrt{K-1}\right)\right]^2 dt$$

Integration yields

$$\int_0^\infty e^2(t)\,dt = \left(\frac{K}{K-1}\right)\left(\frac{e^{-2t}}{4}\right)\left[-1 - \frac{\cos\left(2\sqrt{K-1}\,t + 2\tan^{-1}\sqrt{K-1} - \tan^{-1}(-\sqrt{K-1})\right)}{\sqrt{K}}\right]\Bigg|_0^\infty$$

$$= \frac{K}{4(K-1)}\left[1 + \frac{\cos\left(2\tan^{-1}\sqrt{K-1} - \tan^{-1}(-\sqrt{K-1})\right)}{K}\right]$$

But

$$\cos\left(2\tan^{-1}\sqrt{K-1}-\tan^{-1}(-\sqrt{K-1})\right)=-\cos 3\sqrt{K-1}=3\cos\sqrt{K-1}-4\cos^3\sqrt{K-1}$$
$$=\frac{3K-4}{K\sqrt{K}}$$

Therefore

$$\int_0^\infty e^2(t)\,dt=\frac{K}{4(K-1)}\left(1+\frac{3K-4}{K^2}\right)=\frac{K}{4(K-1)}\frac{(K-1)(K+4)}{K^2}=\frac{K+4}{4K}$$

The first derivative of $\int_0^\infty e^2(t)\,dt$ with respect to K is given by

$$\frac{d}{dK}\left(\frac{K+4}{4K}\right)=-\frac{1}{K^2}$$

Apparently, $\int_0^\infty e^2(t)\,dt$ decreases monotonically as K increases. Therefore the optimal value of K is $K=\infty$, which is of course unrealizable. For this value of K,

$$\lim_{K\to\infty}\int_0^\infty e^2(t)\,dt=\lim_{K\to\infty}\left(\frac{K+4}{4K}\right)=\frac{1}{4}$$

Note also that the natural frequency ω_n of the optimal system is $\omega_n=\sqrt{K}=\infty$ and the damping ratio $\xi=1/\omega_n=0$, making it marginally stable. Therefore only a *suboptimal* (less than optimal) system can be practically realized and its design depends on the specific application.

Typical optimal control problems, however, are much more complex than this simple example and they require more sophisticated mathematical techniques for their solution. We do little more here than mention their existence.

5. State Estimators

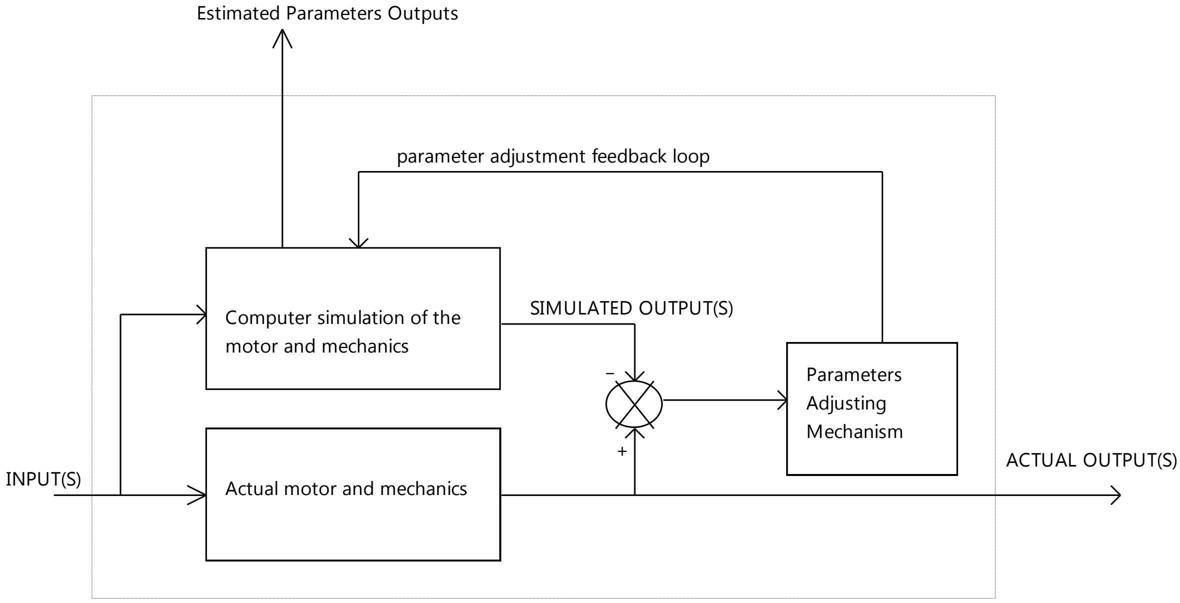
Full order observers: to run a real time simulation of the plant in parallel with the hardware, and obtain the system parameters which cannot be measured. Use these parameters to control the hardware.

Examples of observers are:

- Torque observers
- Velocity observers
- Position observers (sensorless position control)
- Flux observers

Notes:

1. We must monitor the input of the plant, as well as the output of the plant.
2. If the simulated output does not correspond with the actual output, a feedback mechanism must be used to adjust the simulated parameters.
3. The sampling rate of the feedback mechanism is related to the rate of change of the parameter variations.
4. There must be a one-to-one correspondence between input/output relationships and the system parameter changes.
5. The feedback loop must be globally stable.



Design of Full Order Observers for DC Motor Control

A full order observer estimates all states of a plant from the measurement of input(s) and some measurable outputs of that plant. For instance, in the following example, our observer model will use the input signal to a motor as well as the measured position to estimate the velocity and position .

The lower half of figure 3 is the state space presentation of an actual motor, whereas the top part shows an observer model implemented in software. Notice that x_e is the estimated state and Matrix G weights the error, in order to provide a corrective action on the estimation mechanism.

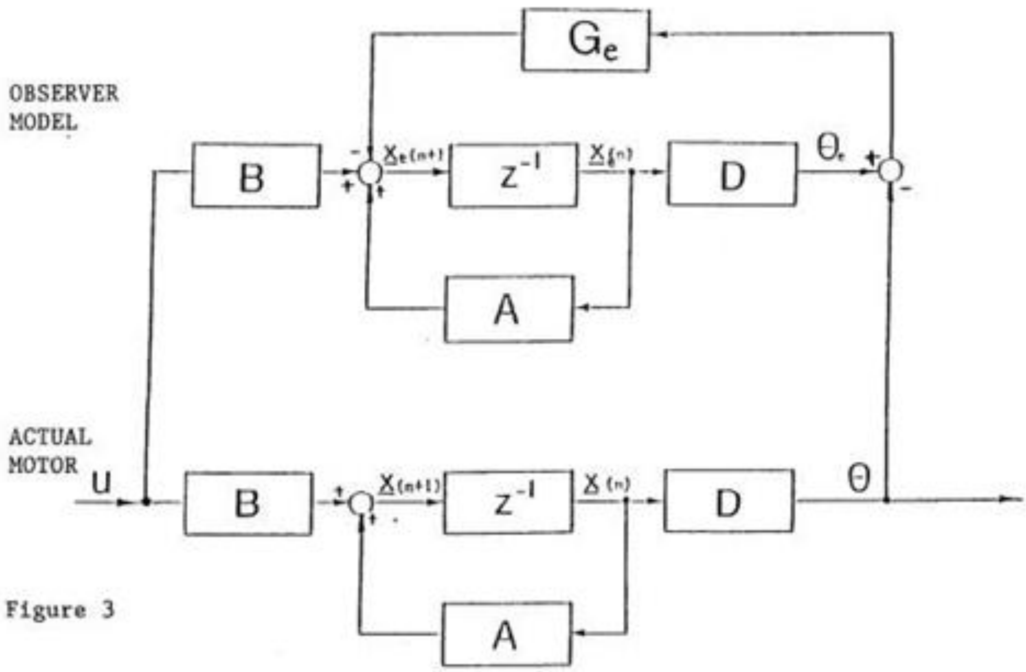


Figure 3

From this block diagram, the observer model may be derived as follows:

6. Adaptive Control

Adaptive Control characteristics

Adaptive control involves the control of a plant that involves changing characteristics. Two commonly used configurations are: Self Tuning Regulator and Model Reference Adaptive Control

CONTROLLER DESIGN

The purpose of an identification algorithm such as we have just described is to provide a controller design-block with appropriate values of system parameters. The controller will use these to calculate a new set of control parameters. The actual design of the controller will depend upon what sort of performance is desired for a particular application.

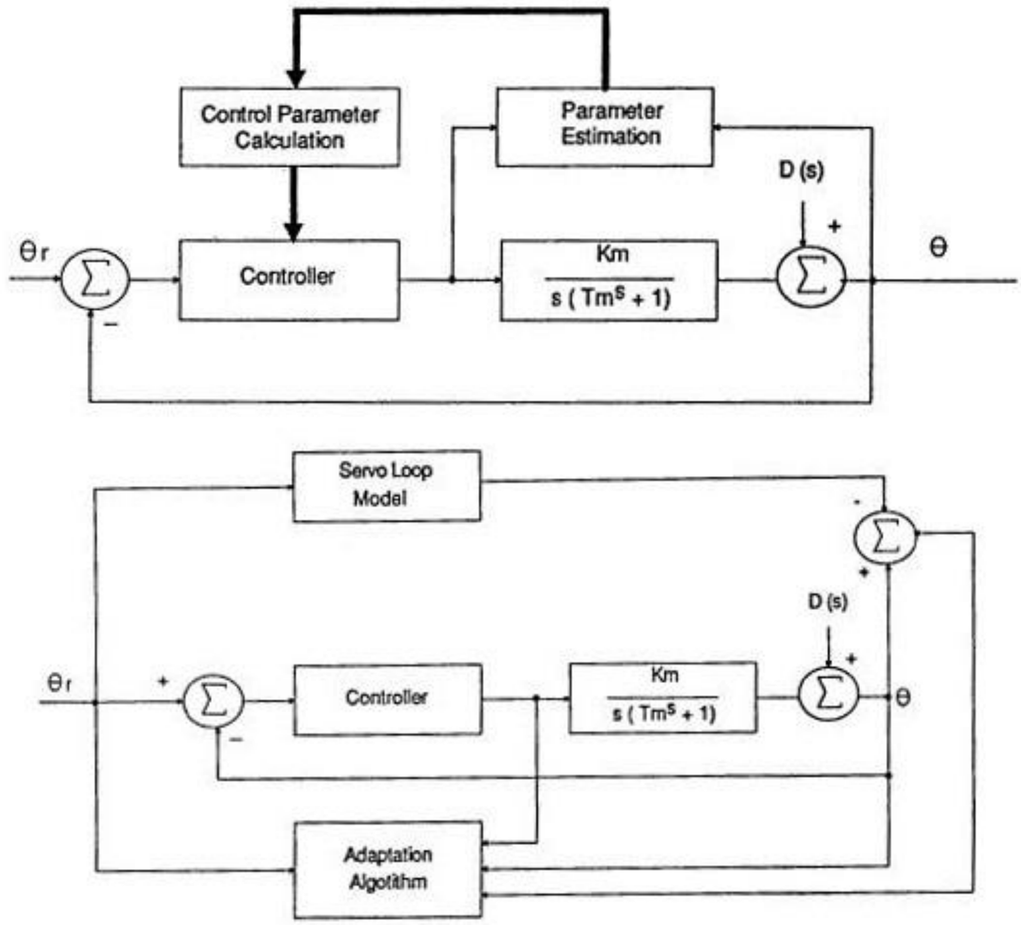


Figure 15: Model Reference Adaptive Control

Adaptive Control versus Robust Control

The adaptive controller adapt to changes of the plant.

Robust control is robust enough to allow variations of system parameters in the plant.

ADAPTIVE CONTROL

An adaptive controller is one which adjusts one or more coefficients of the control law in such a way that the resulting closed loop performance does not suffer with changing parameters and load coefficients. The two broad categories of adaptive control are Model Reference Adaptive Control (MRAC) and Self-Tuning Regulator (STR). Early work in the field of adaptive control was done by Kalman in 1958 [1] in which he compared the self-adaptive controller with that of the control system's designer. The STR concept was introduced over a decade ago [2] and implemented later [3] [4].

The STR control strategy, as shown in Figure 14, is based on estimating the system parameters and adjusting the control setting accordingly. The MRAC cannot be described by this structure and is shown in Figure 15. MRAC involves the comparison between a reference model and the actual system, which may be the motor only or the entire position or velocity loop. The comparison of reference to motor is categorized in [5] as explicit identification, implicit control and the servo loop comparison systems are categorized as implicit identification, explicit control. Self tuning and MRAC techniques are based on different design principles but in some special cases [6], [7] the control schemes are very similar. Both schemes make the resulting system nonlinear and thus require careful stability analysis. Each control strategy may be implemented in several configurations. The MRAC system, however, will require model calculations and a parameter identification scheme. This and other practical considerations will assist the designer in selecting and implementing either MRAC or STR, rather than strictly theoritical concerns.

ROBUST CONTROL

The MRAC and STR concepts we have discussed involve changing coefficients of the controller transfer function as the system operates in real time. Robust Control, in Contrast, seeks to make the overall closed loop transfer function independent of plant parameter variations and disturbances without real-time modification of the control scheme. A robust controller accomplishes this by utilizing a carefully designed controller with high loop gain so that unknown and unpredictable changes in the plant are handled in a uniform manner. Also, because there are no slow-responding parameter identification algorithm, quite rapid changes in plant parameters can be tolerated. Further, plant parameter variations and disturbances are handled by the same mechanism so that robust controller works well in eliminating the effects of either.

The benefits of robust control can be obtained through the use of high loop gain or bandwidth. This causes the problems of sensor noise and unmodeled system dynamics to be the ultimate limitations of the robust control technique. While traditional theory would suggest that high loop gain is sufficient to obtain robustness of the motor and load combination, practical problems discourage the use of a brute force high gain approach. The greatest problem is that the system has lightly damped complex poles usually due to torsional resonances. These poles cause stability problems at high loop gains. For this reason, a robust controller will require accurate system modeling and a careful compensation design.

Glossary – English/Chinese Translation

English	Chinese
robust control	稳健的控制
feedforward compensation	前馈补偿
controllability and observability	可控性和可观察性
state feedback control	状态反馈控制
state observer	国家观察员
state estimator	状态估计器
adaptive control	自适应控制
self tuning regulator	自整定稳压器
friction model	摩擦模型
optimal control	最佳控制
performance index	性能指标
real time simulation	实时模拟
model reference adaptive control	型号参考自适应控制