The gain margin, measured at $\omega_\pi = 5$ rad/s, is 20 db. Thus the Bode gain can be raised by as much as $20 - 6 = 14$ db and still satisfy the gain margin requirement. However, the Bode phase angle plot indicates that, for $\phi_{PM} \geqq 45°$, the gain crossover frequency ω_1 must be less than about 2 rad/s. The magnitude curve can be raised by as much as 7.5 db before ω_1 exceeds 2 rad/s. Thus the maximum value of K_B satisfying *both* specifications is 7.5 db, or 2.37.

Question 2 – lead compensation

Design compensation for the system

$$GH(j\omega) = \frac{8}{(1+j\omega)(1+j\omega/3)^2}$$

which will yield an overall phase margin of 45° and the same gain crossover frequency ω_1 as the uncompensated system. The latter is essentially the same as designing for

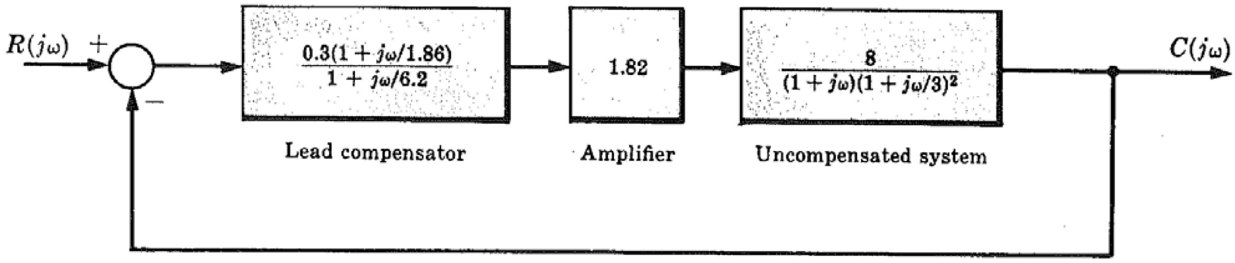


Fig. 16-14

The Bode plots for the uncompensated system are shown in Fig. 16-12(*a*) and (*b*).

The gain crossover frequency ω_1 is 3.4 rad/s and the phase margin is 10°. The specifications can be met with a cascade lead compensator and gain factor amplifier. Choosing a and b for the lead compensator is somewhat arbitrary, as long as the phase lead at $\omega_1 = 3.4$ is sufficient to raise the phase margin from 10° to 45°. However, it is often desirable, for economic reasons, to minimize the low frequency attenuation obtained from the lead network by choosing the largest lead ratio $a/b < 1$ that will supply the required amount of phase lead. Assuming this is the case, the maximum lead ratio that will yield $45° - 10° = 35°$ phase lead is about 0.3 from Fig. 16-2. Solution

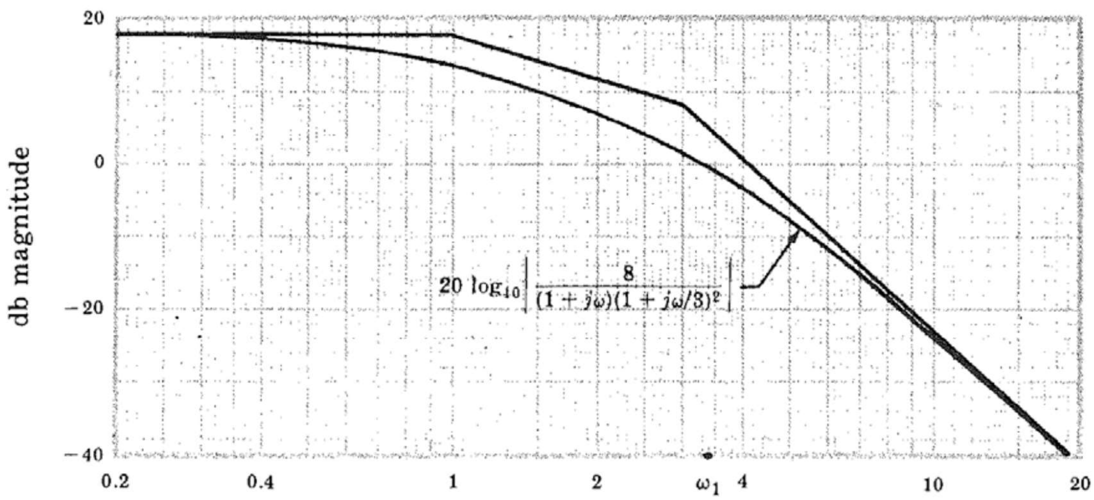


Fig. 16-12(*a*)

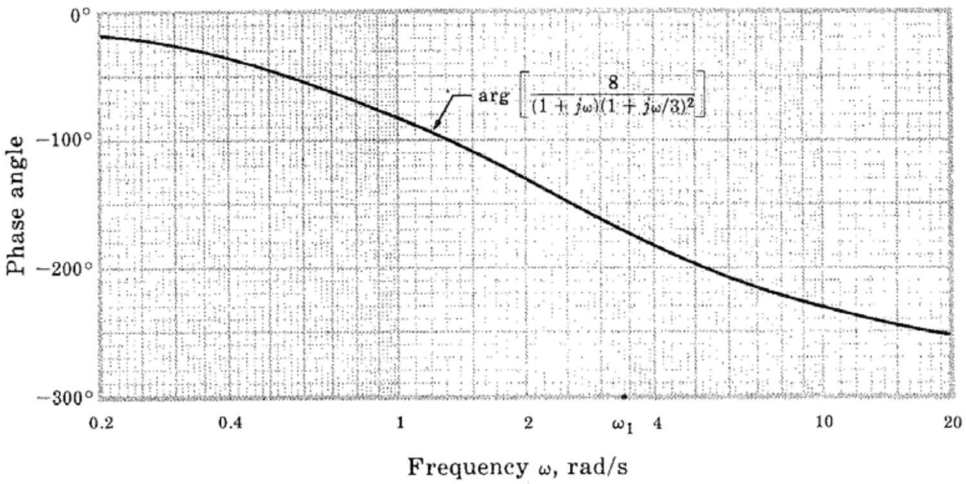
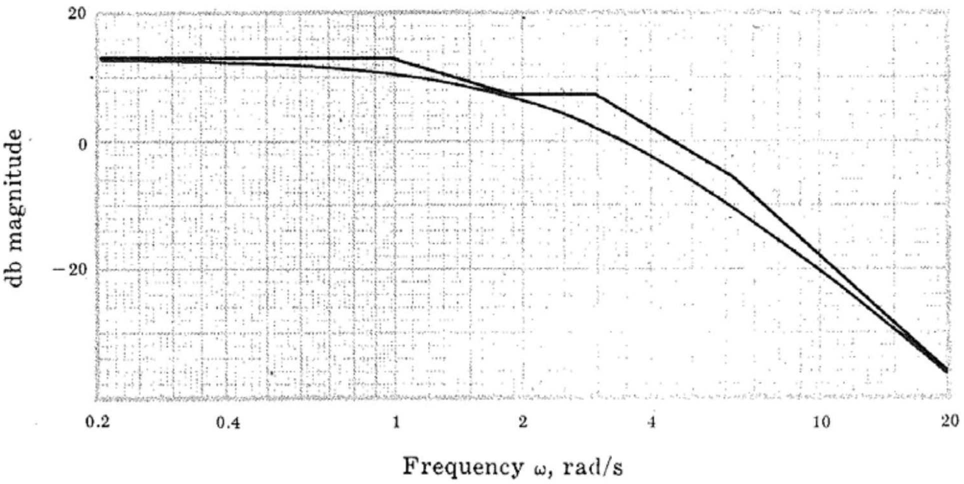
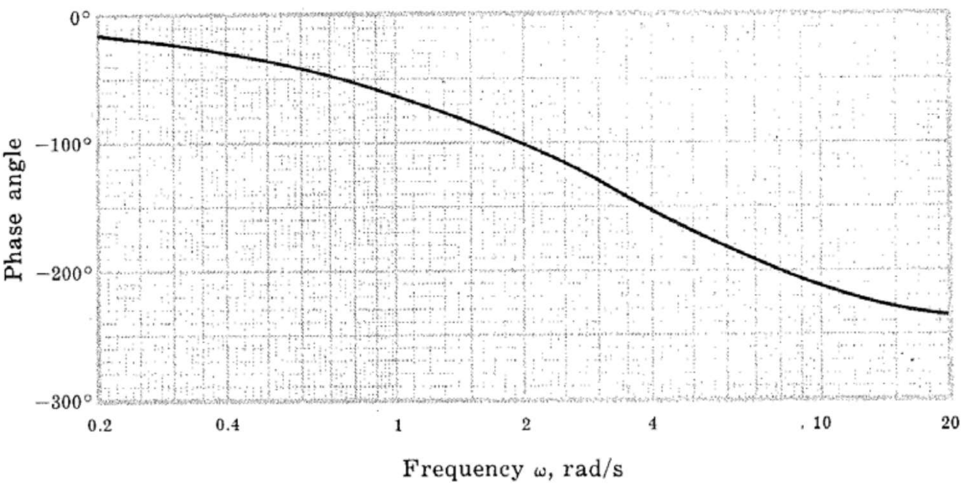


Fig. 16-12(*b*)

of equation (*16.2*) yields a value of $a/b = 0.27$. But we shall use $a/b = 0.3$ because we have the curves available for this value in Fig. 16-2. We want to choose a and b such that the maximum phase lead, which occurs at $\omega_m = \sqrt{ab}$, is obtained at $\omega_1 = 3.4$ rad/s. Thus, $\sqrt{ab} = 3.4$. Substituting $a = 0.3b$ into this equation and solving for b, we find $b = 6.2$ and $a = 1.86$. But this compensator produces $20 \log_{10} \sqrt{6.2/1.86} = 5.2$ db attenuation at $\omega_1 = 3.4$ rad/s (see Problem 16.4). Thus an amplifier with a gain of 5.2 db, or 1.82, is required, in addition to the lead compensator, to maintain ω_1 at 3.4 rad/s. The Bode plots for the compensated system are shown in Fig. 16-13 and the block diagram in Fig. 16-14 below.





Question 3 – lag compensation

Design a unity-feedback system, with the fixed plant

$$G_2(j\omega) = \frac{1}{(1+j\omega/3)^3}$$

satisfying the specifications: (1) $K_p \geqq 4$, (2) gain margin $\geqq$ 12 db, (3) phase margin $\geqq 45°$.

The specification on the position error constant K_p requires a Bode gain increase by a factor of 4. The Bode plots for this system, with the gain increased by $20 \log_{10} 4 = 12$ db, are shown in Fig. 16-16.

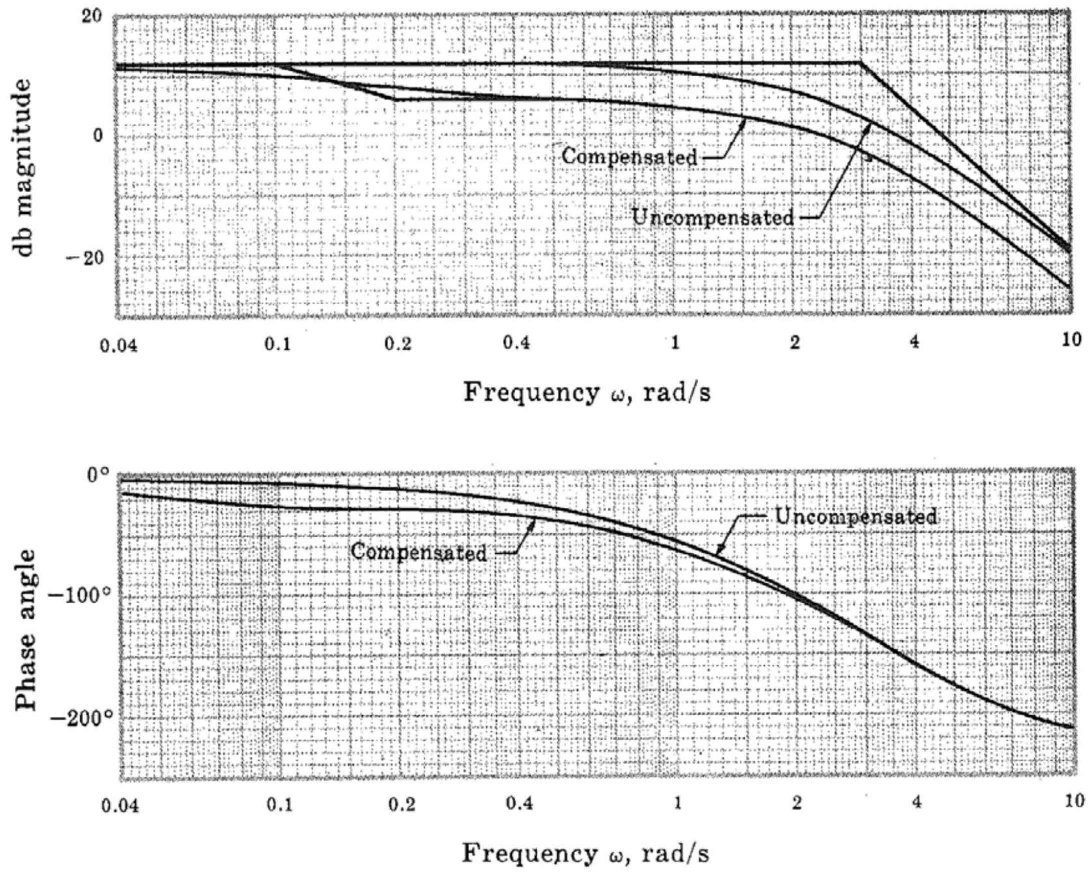


Fig. 16-16

The gain margin is 6 db and the phase margin is 30°. These margins can be increased by adding a lag compensator. To get the gain margin up to 12 db, the high frequency magnitude must be reduced by 6 db. To raise the phase margin to 45°, ω_1 must be lowered to 3.0 rad/s or less. This requires a magnitude attenuation of 3 db at that frequency. Therefore let us choose a lag ratio $b/a = 2$ to yield a high frequency attenuation of $20 \log_{10} 2 = 6$ db. For $a = 0.1$ and $b = 0.2$ the phase margin is 65° and the gain margin is 12 db, as shown in the compensated Bode plots of Fig. 16-16.

The compensated open-loop frequency response function is $\dfrac{4(1+j\omega/0.2)}{(1+j\omega/0.1)(1+j\omega/3)^3}$.