Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 01-k

Bode Design and Compensation

Contents

Reference:

Chapter 8 "Frequency Response Analysis; K. Ogata, "Modern Control Engineering"

Chapter 15 & 16, "Feedback and Control Systems" Schaum's Outline Series, McGraw Hill.

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1. Introduction

The three predominant objectives of feedback control systems analysis are the determination of the following system characteristics:

1. The degree or extent of system stability
2. The steady-state performance
3. The transient response

Knowing whether a system is absolutely stable or not is insufficient information for most purposes. If a system is stable, we usually want to know how close the system is to being unstable. We need to determine its *relative stability.*

Unity Feedback System

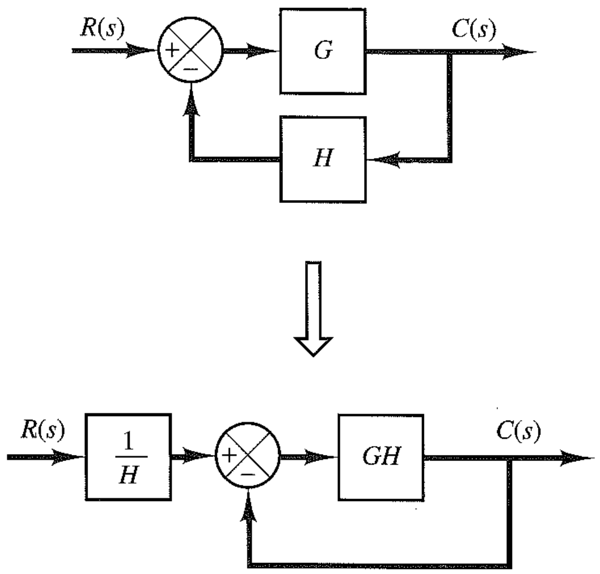


Figure 8–70
Modification of a system with feedback elements to a unity-feedback system.

a unity-feedback system, as shown in Figure 8–70. Hence, the extension of relative stability analysis for the unity-feedback system to nonunity-feedback systems is possible.

1. Gain Margin

Gain margin, a measure of relative stability, is defined as the magnitude of the reciprocal of the open-loop transfer function, evaluated at the frequency ω_π at which the phase angle (see Chapter 6) is −180 degrees. That is,

$$\text{gain margin} \equiv \frac{1}{|GH(j\omega_\pi)|} \qquad (10.1)$$

where $\arg GH(j\omega_\pi) = -180 \text{ degrees} = -\pi \text{ radians}$ and ω_π is called the **phase crossover frequency.**

2. Phase Margin ϕ_{PM}

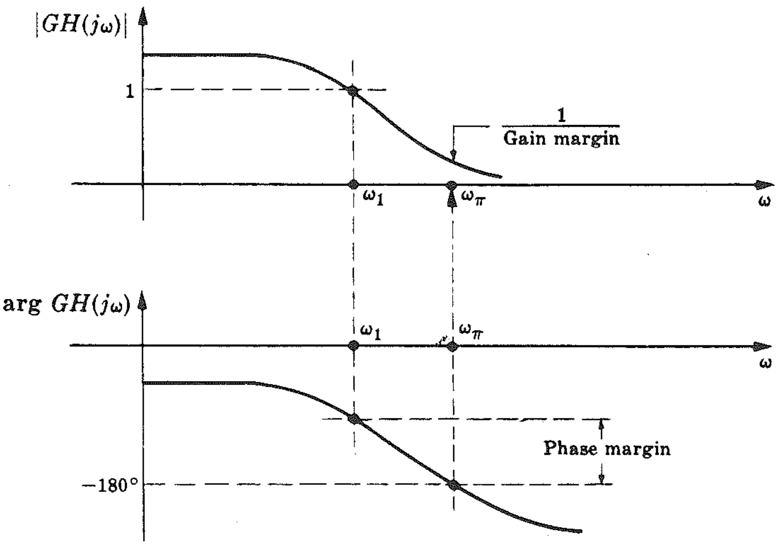
Phase margin ϕ_{PM}, a measure of relative stability, is defined as 180 degrees plus the phase angle ϕ_1 of the open-loop transfer function at unity gain. That is,

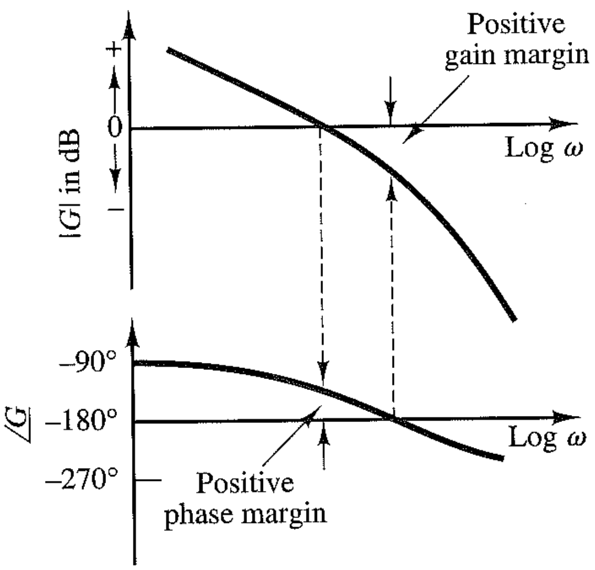
$$\phi_{PM} \equiv [180 + \arg GH(j\omega_1)] \text{ degrees} \qquad (10.2)$$

where $|GH(j\omega_1)| = 1$ and ω_1 is called the **gain crossover frequency.**

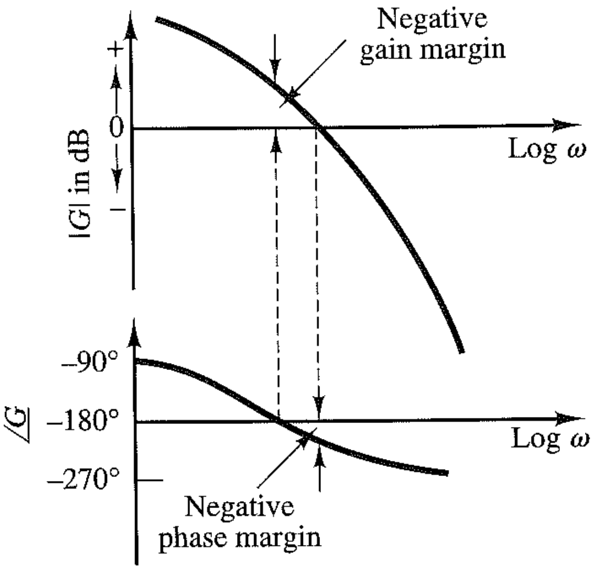
Example 10.1.

The gain and phase margins of a typical feedback control system are illustrated in the following graphs:





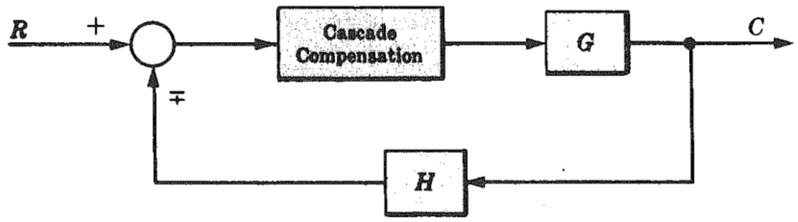
Stable system

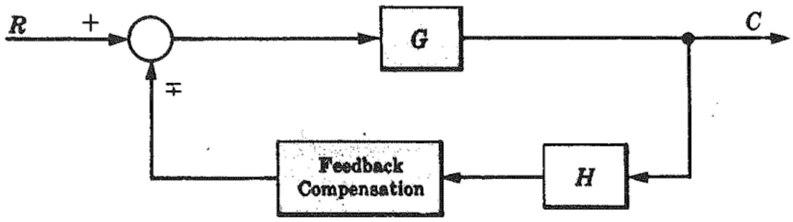


Unstable system

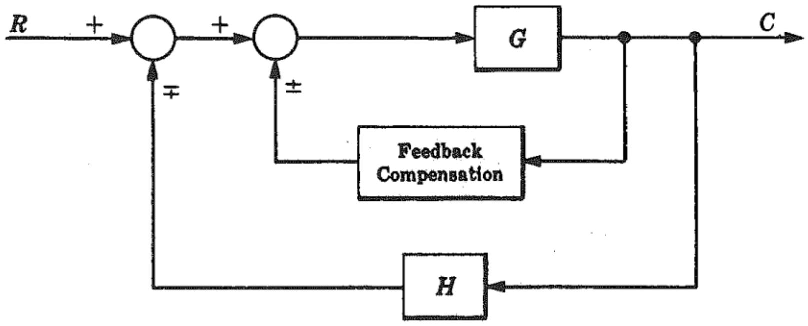
2. Compensator Design Methods

In order to meet the performance specifications for feedback control systems, appropriate *compensation* networks must usually be introduced into the system. (We assume here that G and H are fixed configurations of components over which the designer has no control.) Compensation networks may consist of either passive or active elements, several of which have already been discussed, especially in Chapter 6. They may be introduced into the forward path (cascade compensation), or the feedback path (feedback compensation):





Feedback compensation may also occur in minor feedback loops:



3. Bode Design Philosophy

Design of a feedback control system using Bode techniques entails shaping and reshaping the Bode magnitude and phase angle plots until the system specifications are satisfied. These specifications are most conveniently expressed in terms of frequency-domain figures of merit such as gain and phase margin for the transient performance and the error constants (Chapter 9) for the steady-state time-domain response.

Shaping the asymptotic Bode plots by adding cascade or feedback compensation is a relatively simple procedure. Bode plots for several common compensation networks are presented in Sections 16.3, 16.4, and 16.5. With these graphs, the magnitude and phase angle contributions of a particular compensator can be added directly to the uncompensated system Bode plots.

4. Gain Factor Compensation

It is possible in some cases to satisfy all system specifications by simply adjusting the open-loop gain-factor K. But, when working with Bode plots, it is more convenient to use the Bode gain:

$$K_B = \frac{K \prod_{i=1}^{m} z_i}{\prod_{i=1}^{n} p_i}$$

where $-p_i$ and $-z_i$ are the finite poles and zeros of GH.

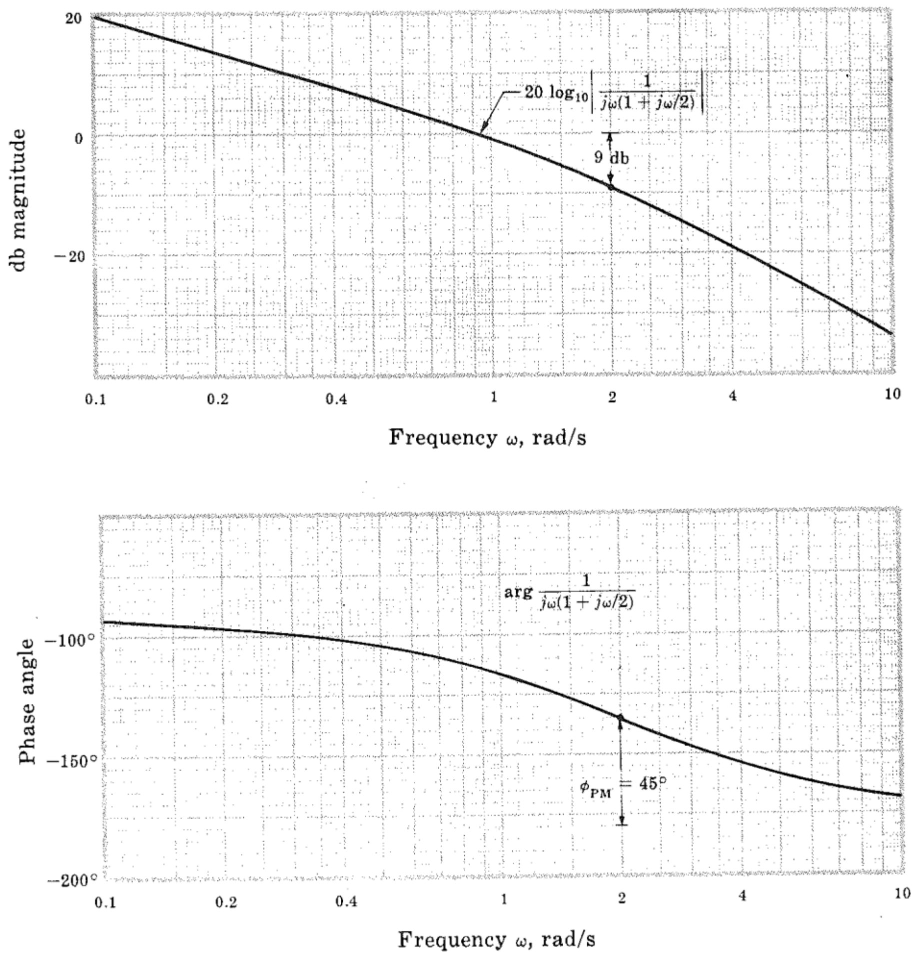
Example 16.1.

The Bode plots for

$$GH(j\omega) = \frac{K_B}{j\omega(1 + j\omega/2)}$$

are shown in Fig. 16-1 for $K_B = 1$.

The maximum amount K_B may be increased to improve the system steady-state performance without decreasing the phase margin below 45° is determined as follows. In Fig. 16-1 below, the phase margin is 45° if the gain crossover frequency ω_1 is 2 rad/s and the magnitude plot can be raised by as much as 9 db before ω_1 becomes 2 rad/s. Thus K_B can be increased by 9 db without decreasing the phase margin below 45°.



5. Lead Compensation

The lead compensator, presented in Sections 6.3 and 12.4, has the following Bode form frequency response function:

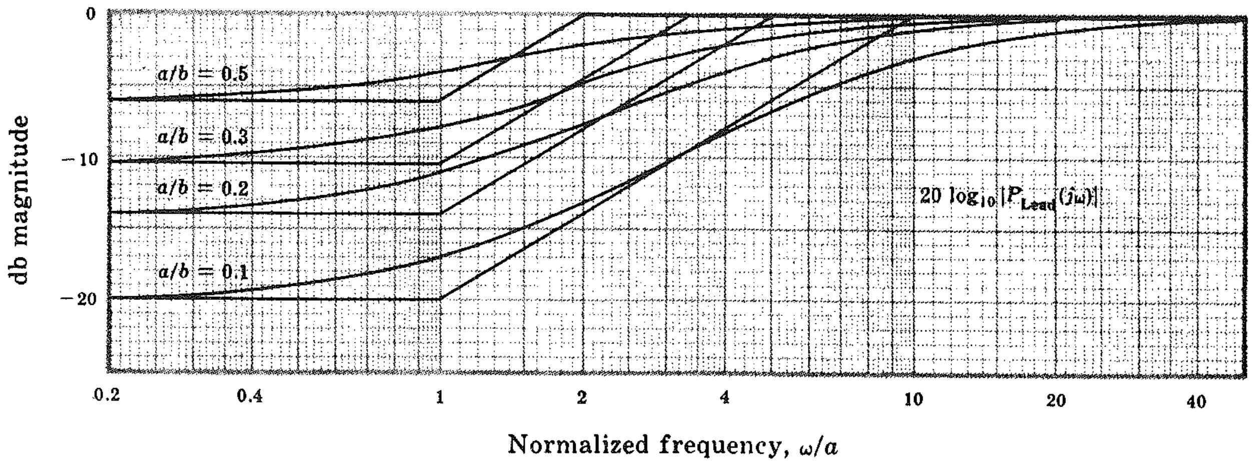
$$P_{\text{Lead}}(j\omega) \quad = \quad \frac{\frac{a}{b}(1 + j\omega/a)}{1 + j\omega/b} \qquad (16.1)$$

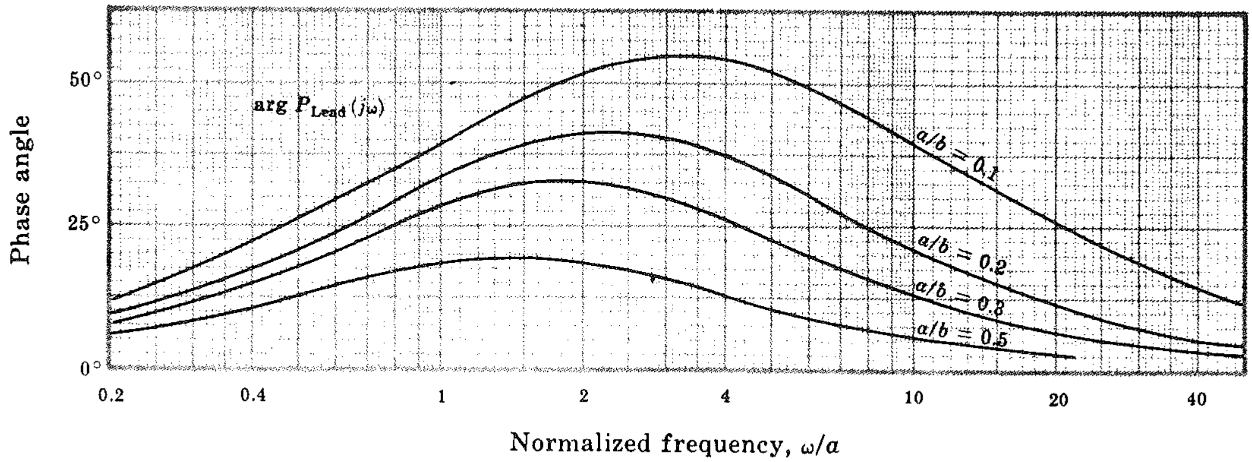
The Bode plots for this compensator, for various *lead ratios* a/b, are shown in Fig. 16-2 below. These graphs illustrate that addition of a cascade lead compensator to a system lowers the overall magnitude curve in the low frequency region and raises the overall phase angle curve in the low to mid-frequency region. Other properties of the lead compensator are discussed in Section 12.4, page 226.

The amount of low frequency attenuation and phase lead produced by the lead compensator depends on the lead ratio a/b. Maximum phase lead occurs at the frequency $\omega_m = \sqrt{ab}$ and is equal to

$$\phi_{\max} \quad = \quad (90 - 2\tan^{-1}\sqrt{a/b}\,)\ \text{degrees} \qquad (16.2)$$

Lead compensation is normally used to increase the gain and/or phase margins of a system or increase its bandwidth. Additional modification of the Bode gain K_B is usually required with lead networks, as described in Section 12.4.





6. Lag Compensation

$$P_{\text{Lag}}(j\omega) = \frac{1 + j\omega/b}{1 + j\omega/a} \qquad (16.3)$$

The Bode plots for the lag compensator, for various *lag ratios* b/a, are shown in Fig. 16-6. The properties of this compensator are discussed in Section 12.5, page 228.

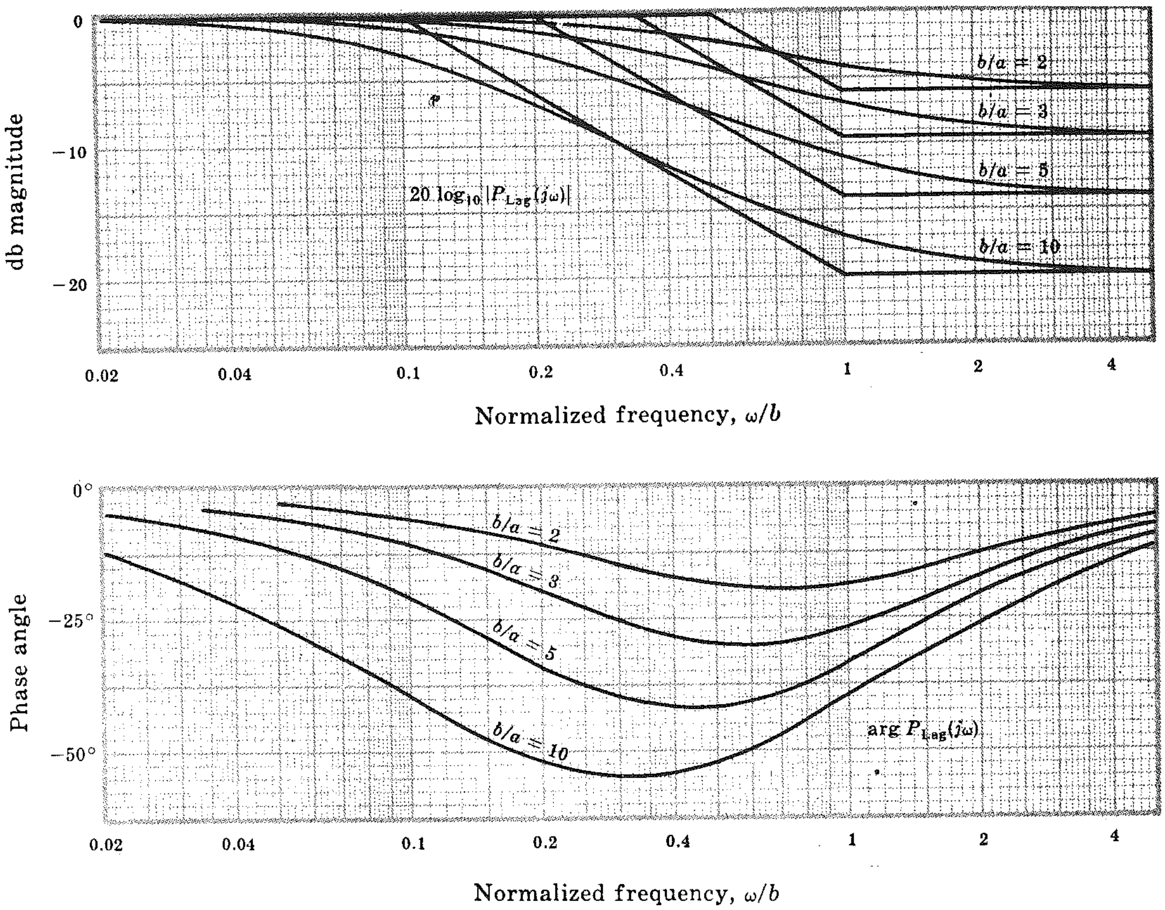


Fig. 16-6

7. Lead-Lag Compensation

It is sometimes desirable, as discussed in Section 12.6, page 230, to simultaneously employ both lead and lag compensation. Although one each of these two networks can be connected in series to achieve the desired effect, it is usually more convenient to mechanize the combined lag-lead compensator described in Example 6.6, page 99. This compensator can be constructed with a single RC network, as shown in Problem 6.14, page 103.

The Bode form of the frequency response function for the lag-lead compensator is

$$P_{LL}(j\omega) = \frac{(1+j\omega/a_1)(1+j\omega/b_2)}{(1+j\omega/b_1)(1+j\omega/a_2)}$$

with $b_1 > a_1$, $b_2 > a_2$ and $a_1 b_2 = b_1 a_2$. A typical Bode magnitude plot in which $a_1 > b_2$ is shown in Fig. 16-8. The Bode plots for a specific lag-lead compensator can be determined by combining the Bode plots for the lag portion from Fig. 16-6 with those for the lead portion from Fig. 16-2. Additional properties of the lag-lead compensator are discussed in Section 12.6.

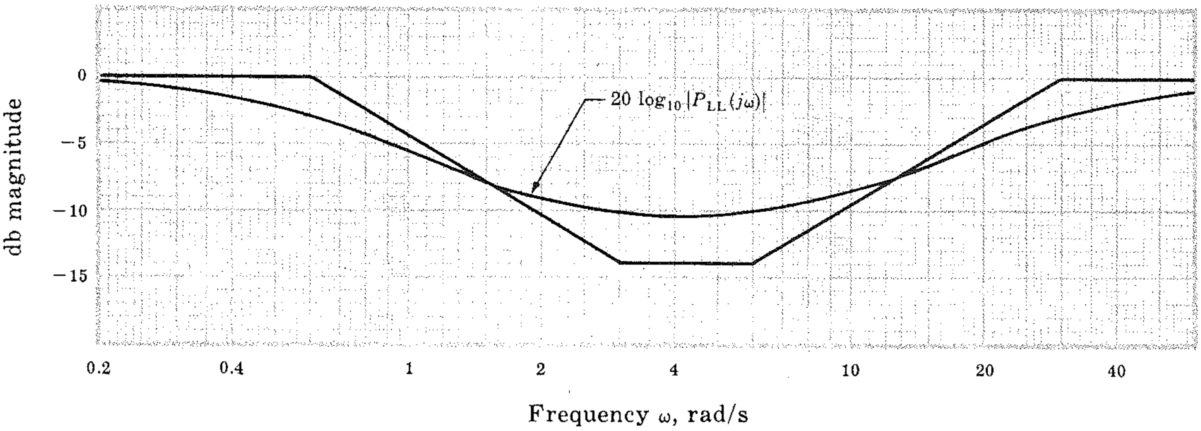


Fig. 16-8

8. Design Examples

In the first approach, the high-frequency portion of $G_p(s)$ is rotated in the counterclockwise direction, which means that more phase is added to the system in the positive direction in the proper frequency range. This scheme is basically referred to as **phase-lead compensation**, and controllers used for this purpose are often of the high-pass-filter type. The second approach apparently involves the shifting of the low-frequency part of the $K = 1.2$ trajectory in the clockwise direction, or alternatively, reducing the magnitude of $G_p(s)$ with $K = 100$ at the high-frequency range. This scheme is often referred to as the **phase-lag compensation**, since negative phase or phase lag is introduced to the system in the low-frequency range. The type of controllers used for the phase-lag compensation is often of the low-pass filter type.

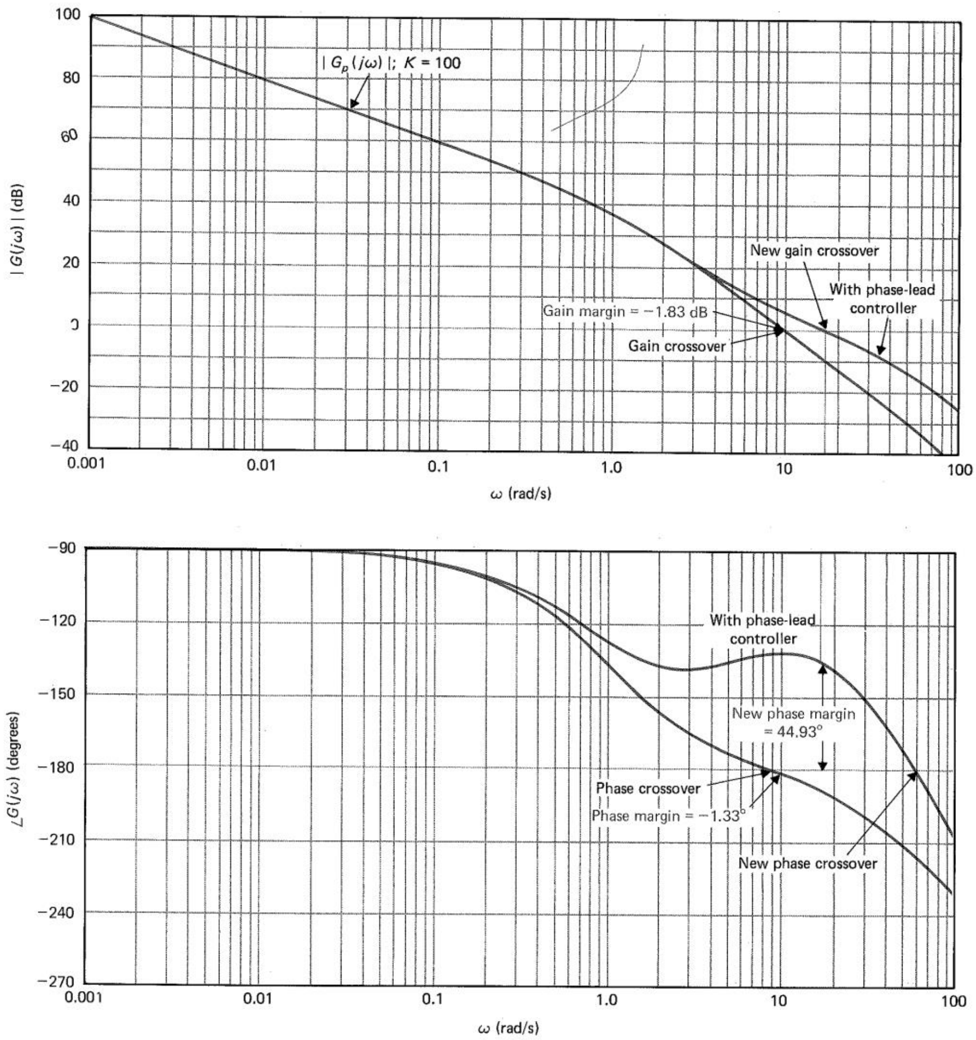


FIGURE 11-2 Bode diagram of $G_p(s) = 100/[s(1 + s)(1 + 0.0125s)]$ with phase-lead compensation.

Figures 11-2 and 11-3 further illustrate the philosophy of design in the frequency domain using the Bode diagram. In this case, the relative stability of the system is more conveniently represented by the gain margin and the phase margin. In Fig. 11-2, the Bode plot of $G_p(j\omega)$ shows that when $K = 100$, the gain and phase margins are both negative, and the system is unstable. By using the first approach, the phase-lead compensation, as described earlier, more positive phase is added to $G_p(j\omega)$ at the high frequencies to improve the phase margin. It should be noted that positive phase in the high-pass filter is accompanied by an additional gain at the high frequencies that tends to push up the gain-crossover frequency. In general, depending on the characteristics of the uncompensated system, if the design is carried out properly, it is possible to obtain a net improvement in relative stability using this approach. The Bode diagram in Fig. 11-3 serves to illustrate the principle of phase-lag compensation. If, instead of adding more positive phase to $G_p(j\omega)$ in the high-frequency range, as in Fig. 11-2, we attenuate the magnitude of $G_p(j\omega)$, with $K = 100$, in the high-frequency range by means of a low-pass filter, a similar stabilization effect can be achieved. The Bode diagram of Fig. 11-3 shows that if the attenuation starts at a sufficiently low-frequency range, the effect on the phase of $G_p(j\omega)$ due to the phase-lag compensation is insignificant at the new gain-crossover frequency. Thus, the net effect of the compensation scheme is the improvement of the relative stability of the system.

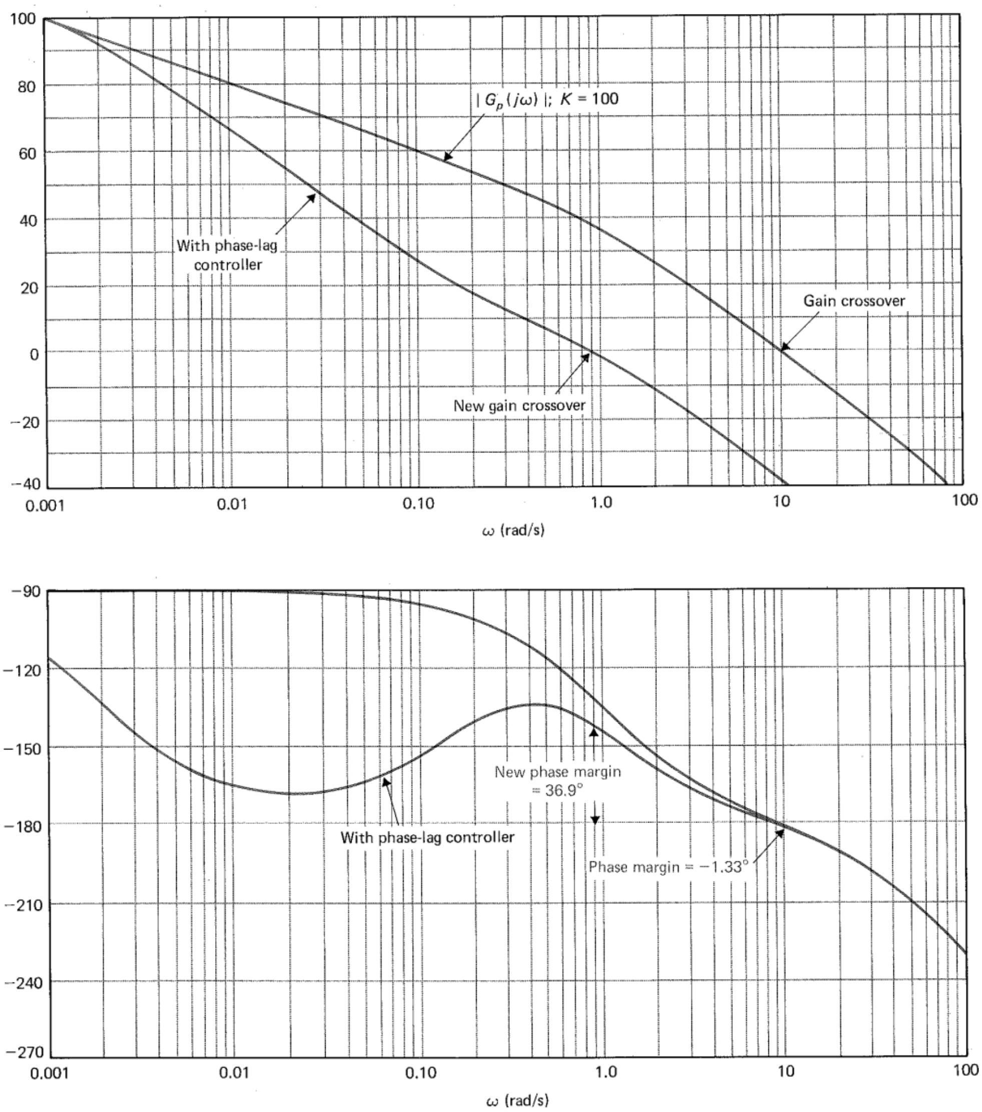


FIGURE 11-3 Bode diagram of $G_p(s) = 100/[s(1 + s)(1 + 0.0125s)]$ with phase-lag compensation.

The examples given are simply for the purpose of illustrating the principle of design of control systems in the frequency domain using controllers with phase-lead and phase-lag characteristics. In general, it may not be possible to satisfy all design criteria by simply using a phase-lead or a phase-lag controller. Situations often arise that both the low-frequency and the high-frequency loci of the process may need to be reshaped by using a controller with lead-lag or lag-lead characteristics. Or, the frequency locus of the controlled process must be reshaped in a certain midfrequency range so that a so-called "notch" controller such as the bridged-T network (see Section 11-8) is required.

It should be pointed out that the design of linear control systems in the frequency domain is not an exact science. For a given set of performance specifications, a number of controller schemes and parameter values may satisfy. Some degree of trial and error and fine tuning is inevitable. Therefore, a computer program for frequency-domain analysis, such as the FREQRP of the ACSP software package, should be invaluable for the design of linear control systems in the frequency domain.

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Glossary – English/Chinese Translation

English	Chinese
Bode Plot Design Philosophy	博德地块设计理念
steady state performance	稳态性能
transient response	瞬态响应
relative stability	相对稳定性
gain margin	获得利润
phase margin	相位裕量
gain cross over frequency	增益交叉频率
gain factor compensation	增益因数补偿
phase lead compensation	相位引线补偿
phase lag compensation	相位滞后补偿

Your Notes: