Tutorial – 1-01-j

Q1 Ex 14-6 (dominant pole)

By using the dorminant root approach, find the step response of the following system, based on a reduced order (1^{st} order) model:

(a) $$\frac{C}{R} = \frac{5}{(s+1)(s+5)}$$

(b) $$\frac{C}{R} = \frac{5.5(s+0.91)}{(s+1)(s+5)}$$

Q2 Ex 14-10 (Point Design)

Design a phase compensator G_1, so that for the plant G_2:

$$G_2 = \frac{K}{s(s+2)^2}$$

The closed-loop response must have a 10 to 90% rise time less than 1 sec, and an overshoot less than 20%.

Q3 14-1, 14-2 (Gain Factor Compensation)

(a)

Determine the value of the gain factor K for which the system with the open-loop transfer function

$$GH = \frac{K}{s(s+2)(s+4)}$$

has closed-loop poles with a damping ratio $\zeta = 0.5$.

(b)

Determine a value of K for which the system with the open-loop transfer function

$$GH = \frac{K}{(s+2)^2(s+3)}$$

satisfies the following specifications: (a) $K_p \geq 2$. (b) gain margin ≥ 3.

Solution

Q1

(a)

$$\frac{C}{R}=\frac{5}{(s+1)(s+5)}$$

The step response of this system is

$$c(t)=1-1.25e^{-t}+0.25e^{-5t}$$

The term in the response due to the pole at $s_1=\sigma_1=-5$ decays five times as fast as the term due to the pole at $s_2=\sigma_2=-1$. Furthermore, the residue at the pole at $s_1=-5$ is only $\frac{1}{5}$ that of the one at $s_2=-1$. Therefore for most practical purposes the effect of the pole at $s_1=-5$ can be ignored and the system approximated by

$$\frac{C}{R}\cong\frac{1}{s+1}$$

The pole at $s_1=-5$ has been removed from the transfer function and the numerator has been adjusted to maintain the same steady state gain $((C/R)(0)=1)$. The response of the approximate system is $c(t)=1-e^{-t}$.

(b)

$$\frac{C}{R}=\frac{5.5(s+0.91)}{(s+1)(s+5)}$$

has the step response

$$c(t)=1+0.125e^{-t}-1.125e^{-5t}$$

In this case, the presence of a zero close to the pole at -1 significantly reduces the magnitude of the residue at that pole. Consequently, it is the pole at -5 which now dominates the response of the system. The closed-loop pole and zero effectively cancel each other and $(C/R)(0)=1$ so that an approximate transfer function is

$$\frac{C}{R}\cong\frac{5}{s+5}$$

and the corresponding approximate step response is $c\cong 1-e^{-5t}$.

Q2

EXAMPLE 14.10. Consider the continuous plant

$$G_2=\frac{K}{s(s+2)^2}$$

The closed-loop response must have a 10 to 90% rise time less than 1 sec, and an overshoot less than 20%. We observe from Fig. 3-4 that these specifications are met if the closed-loop system has a dominant two-pole configuration with $\zeta=0.5$ and $\omega_n=2$. Thus p_1 is chosen at $-1+j\sqrt{3}$, which is a solution of

$$p_1^2+2\zeta\omega_n p_1+\omega_n^2=0$$

for $\zeta=0.5$ and $\omega_n=2$. Clearly, $p_1^*=-1-j\sqrt{3}$ is the remaining solution of this quadratic equation. The orientation of p_1 with respect to the poles of G_2 is shown in Fig. 14-16.

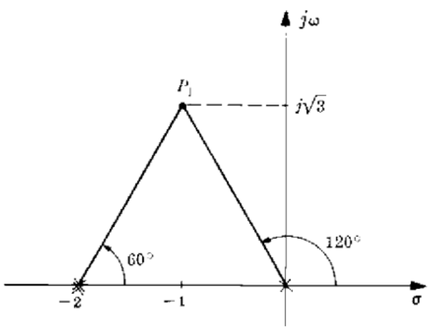

Fig. 14-16

The phase angle of G_2 is $-240°$ at p_1. In order for a branch of the root-locus to pass through p_1, the system must be modified so that the phase angle of the compensated system is $-180°$ at p_1. This can be accomplished by adding a cascade lead network having a phase angle of $240° - 180° = 60°$ at p_1, which is satisfied by

$$G_1 = P_{\text{Lead}} = \frac{s+1}{s+4}$$

as shown in the pole-zero map of the compensated open-loop transfer function G_1G_2 in Fig. 14-17. The closed-loop pole can now be located at p_1 by choosing a value for K which satisfies the root-locus magnitude criterion. Solution of Equation (*13.5*) yields $K = 16$. The root-locus or closed-loop pole-zero map of the compensated system should be sketched to check the validity of the dominant two-pole assumption. Figure 14-18 illustrates that the poles at p_1 and p_1^* dominate the response.

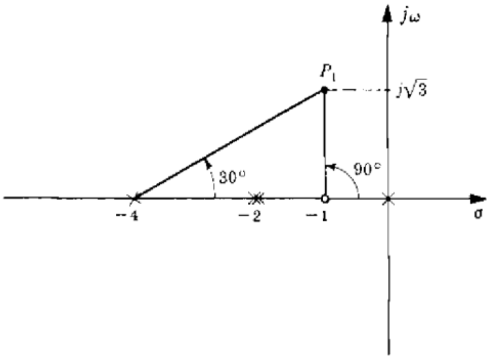

Fig. 14-17

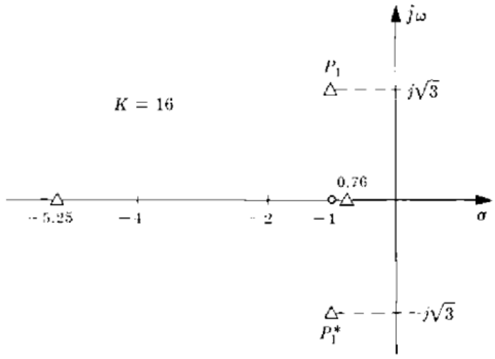

Fig. 14-18

Q3

(a)

$$GH = \frac{K}{s(s+2)(s+4)}$$

has closed-loop poles with a damping ratio $\zeta = 0.5$.

The closed-loop poles will have a damping ratio of 0.5 when they make an angle of 60° with the negative real axis [Equation (*13.18*)]. The desired value of K is determined at the point where the root-locus crosses the $\zeta = 0.5$ line in the s-plane. A sketch of the root-locus is shown in Fig. 14-22. The desired value of K is 8.3.

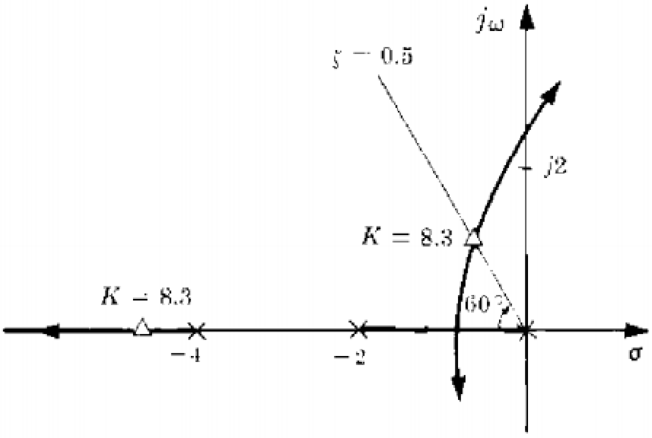

Fig. 14-22

(b)

For this system, K_p is equal to $K/12$. Hence, in order to satisfy the first specification, K must be greater than 24. The value of K at the $j\omega$-axis crossover of the root-locus is equal to 100, as shown in Fig. 14-23. Then, in order to satisfy the second specification, K must be less than $100/3 = 33.3$. A value of K that will satisfy both specifications is 30.

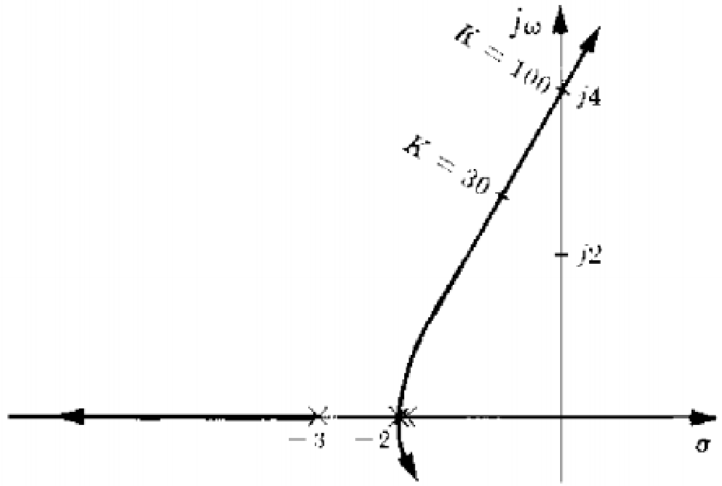

Fig. 14-23