Tutorial – PID control

Q1 (A-10-1)

Describe briefly the dynamic characteristics of the PI controller, PD controller, and PID controller.

Q2 (A-10-2)

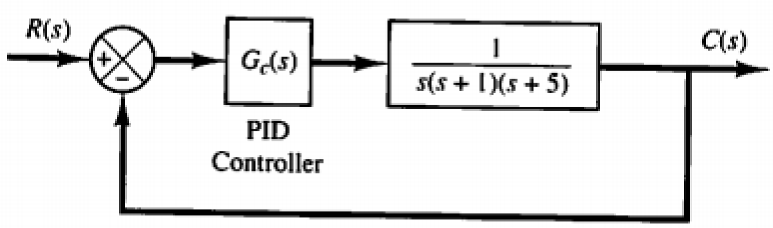
Plot a Bode diagram of a PID controller given by

$$G_c(s) = 2.2 + \frac{2}{s} + 0.2s$$

Q3 (Ex-10-1)

Consider the control system shown in Figure 10–7 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$



Use the Ziegler-Nichols Stability Method to obtain the parameter of the tuning values.

Q4 (EE3005-17-18Q8)

(a) Give 3 reasons why it is difficult to tune the PID values to their optimum values. (5 marks)

(b) Using a step-by-step approach, describe you would tune a PID controller, using the Ziegler and Nichols transient response method. (5 marks)

<u>**Solution**</u>

<u>Q1</u>

Solution. The PI controller is characterized by the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$

The PI controller is a lag compensator. It possesses a zero at $s = -1/T_i$ and a pole at $s = 0$. Thus, the characteristic of the PI controller is infinite gain at zero frequency. This improves the steady-state characteristics. However, inclusion of the PI control action in the system increases the type number of the compensated system by 1, and this causes the compensated system to be less stable or even makes the system unstable. Therefore, the values of K_p and T_i must be chosen carefully to ensure a proper transient response. By properly designing the PI controller, it is possible

to make the transient response to a step input exhibit relatively small or no overshoot. The speed of response, however, becomes much slower. This is because the PI controller, being a low-pass filter, attenuates the high-frequency components of the signal.

The PD controller is a simplified version of the lead compensator. The PD controller has the transfer function $G_c(s)$, where

$$G_c(s) = K_p(1 + T_d s)$$

The value of K_p is usually determined to satisfy the steady-state requirement. The corner frequency $1/T_d$ is chosen such that the phase lead occurs in the neighborhood of the gain crossover frequency. Although the phase margin can be increased, the magnitude of the compensator continues to increase for the frequency region $1/T_d < \omega$. (Thus, the PD controller is a high-pass filter.) Such a continued increase of the magnitude is undesirable, since it amplifies high-frequency noises that may be present in the system. Therefore, lead compensation is preferred over the PD control. Lead compensation can provide a sufficient phase lead, while the increase of the magnitude for the high-frequency region is very much smaller than that for the PD control.

Because the transfer function of the PD controller involves one zero but no pole, it is not possible to electrically realize it by passive *RLC* elements only. Realization of the PD controller using op amps, resistors, and capacitors is possible, but because the PD controller is a high-pass filter, as mentioned earlier, the differentiation process involved may cause serious noise problems in some cases. There is, however, no problem if the PD controller is realized by use of the hydraulic or pneumatic elements.

The PD control, as in the case of the lead compensator, improves the transient-response characteristics, improves system stability, and increases the system bandwidth, which implies fast rise time.

The PID controller is a combination of the PI and PD controllers. It is a lag–lead compensator. Note that the PI control action and PD control action occur in different frequency regions. The PI control action occurs at the low-frequency region and PD control action occurs at the high-frequency region. The PID control may be used when the system requires improvements in both transient and steady-state performances.

Q2

Solution. The controller transfer function $G_c(s)$ can be written as

$$G_c(s) = 2\frac{(0.1s + 1)(s + 1)}{s}$$

Figure 10–23 shows a Bode diagram of the given PID controller.

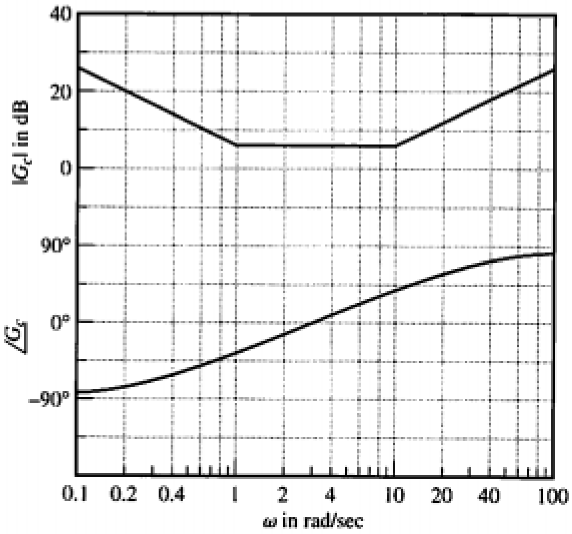


Figure 10–23
Bode diagram of PID controller given by $G_c(s) = 2(0.1s + 1)(s + 1)/s$.

Q3

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s + 1)(s + 5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to K_{cr} (= 30), the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

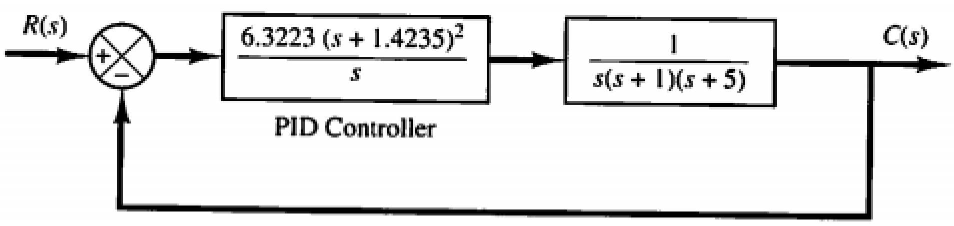
$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Table 10–2, we determine K_p, T_i, and T_d as follows:

$$K_p = 0.6K_{cr} = 18$$
$$T_i = 0.5P_{cr} = 1.405$$
$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right)$$
$$= \frac{6.3223(s + 1.4235)^2}{s}$$



Q4

3 reasons:

1. Do not know when is the most optimal set of values
2. When one is value is adjusted to the best performance, other set values will become less optimal.
3. External disturbance or load change with change the optimal values.

Summary: Transient Response Method

1. Measure the open loop step response of the plant
2. Obtain L and R
3. Calculate the P, I, and D values according to the table below.

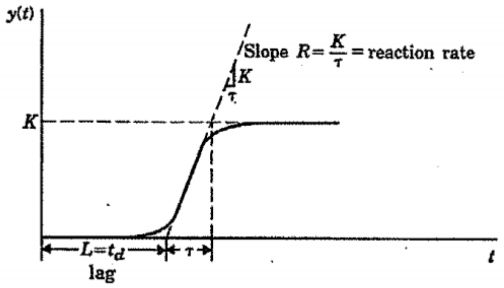


Figure 5.30 Process open-loop step response.

Table 5.2 Ziegler-Nichols tuning parameters using transient response.

	K_p	T_I	T_D
P	$1/RL$		
PI	$0.9/RL$	$3L$	
PID	$1.2/RL$	$2L$	$0.5L$