1-01-g - Tutorial – Root Locus

Question 1

13.1. Determine the closed-loop transfer function and the characteristic equation of the unity negative feedback control system whose open-loop transfer function is $G = K(s+2)/(s+1)(s+4)$.

Question 2

13.15. Find the angles and center of, and sketch the asymptotes for

$$GH = \frac{K(s+2)}{(s+1)(s+3+j)(s+3-j)(s+4)} \qquad K > 0$$

Question 3

13.19. Find the breakaway point for

$$GH = \frac{K(s+2)}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$$

Question 4

13.26. Construct the root-locus for

$$GH = \frac{K}{(s+1)(s+2-j)(s+2+j)} \qquad K > 0$$

Question 5

13.27. Sketch the branches of the root-locus for the transfer function

$$GH = \frac{K(s+2)}{(s+1)(s+3+j)(s+3-j)} \qquad K > 0$$

SOLUTION

Question 1

The closed-loop transfer function is

$$\frac{C}{R} = \frac{G}{1+G} = \frac{K(s+2)}{(s+1)(s+4)+K(s+2)}$$

The characteristic equation is obtained by setting the denominator polynomial equal to zero:

$$(s+1)(s+4)+K(s+2)=0$$

Question 2

$$GH = \frac{K(s+2)}{(s+1)(s+3+j)(s+3-j)(s+4)} \qquad K>0$$

The center of asymptotes is

$$\sigma_c = -\frac{1+3+j+3-j+4-2}{4-1} = -3$$

There are three asymptotes located at angles of $\beta = 60°$, $180°$, and $300°$ as shown in Fig. 13-22.

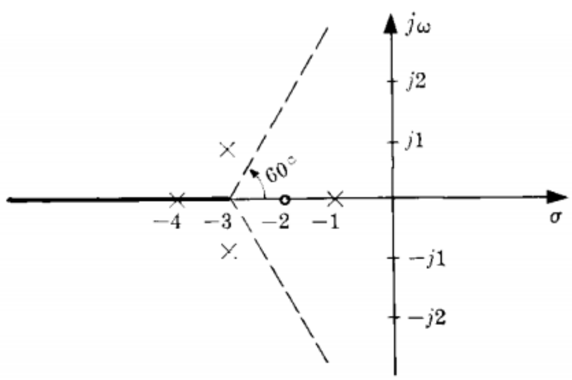

Fig. 13-22

Question 3

$$GH = \frac{K(s+2)}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$$

From Equation (*13.8*),

$$\frac{1}{\sigma_b + 1 + j\sqrt{3}} + \frac{1}{\sigma_b + 1 - j\sqrt{3}} = \frac{1}{\sigma_b + 2}$$

which gives $\sigma_b^2 + 4\sigma_b = 0$. This equation has the solution $\sigma_b = 0$ and $\sigma_b = -4$; $\sigma_b = -4$ is a breakaway point for $K > 0$ and $\sigma_b = 0$ is a breakaway point for $K < 0$, as shown in Fig. 13-25.

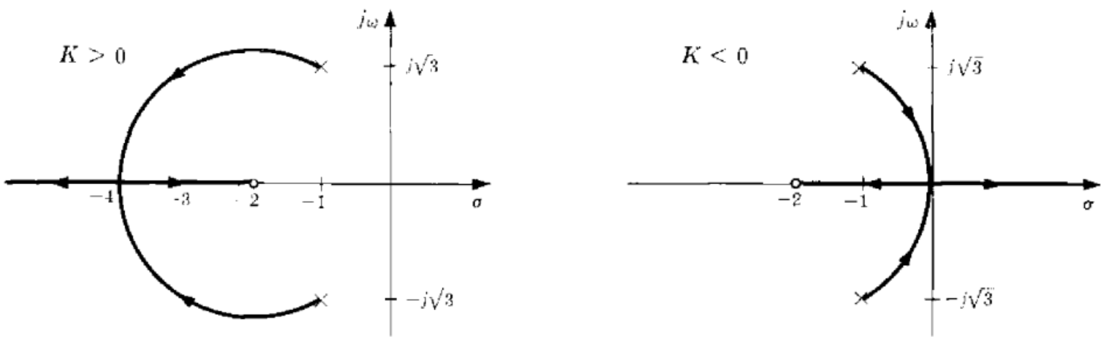

Fig. 13-25

Question 4

$$GH = \frac{K}{(s+1)(s+2-j)(s+2+j)} \qquad K > 0$$

The real axis from -1 to $-\infty$ is on the root-locus. The center of asymptotes is at

$$\sigma_c = \frac{-1-2+j-2-j}{3} = -1.67$$

There are three asymptotes ($n - m = 3$), located at angles of 60°, 180°, and 300°. The departure angle from the complex pole at $s = -2 + j$ computed in Problem 13.24 is $-45°$. A sketch of the resulting root-locus is shown in Fig. 13-28. An accurate root-locus plot is obtained by checking the angle criterion at points along the sketched branches, adjusting the location of the branches if necessary, and then applying the magnitude criterion to determine the values of K at selected points along the branches. The completed root-locus is shown in Fig. 13-29.

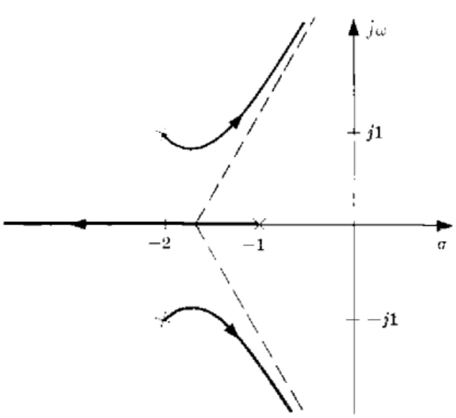

Fig. 13-28

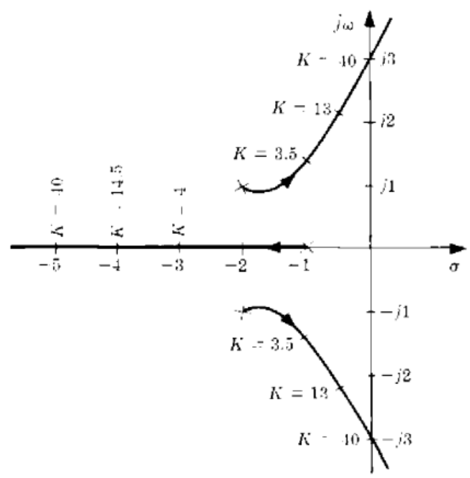

Fig. 13-29

Question 5

$$GH = \frac{K(s+2)}{(s+1)(s+3+j)(s+3-j)} \qquad K > 0$$

The real axis between -1 and -2 is on the root-locus (Problem 13.11). There are two asymptotes with angles of 90° and 270°. The center of asymptotes is easily computed as $\sigma_c = -2.5$ and the departure angle from the complex pole at $s = -3 + j$ as 72°. By symmetry, the departure angle from the pole at $-3 - j$ is $-72°$. The branches of the root-locus may therefore be sketched as shown in Fig. 13-30.

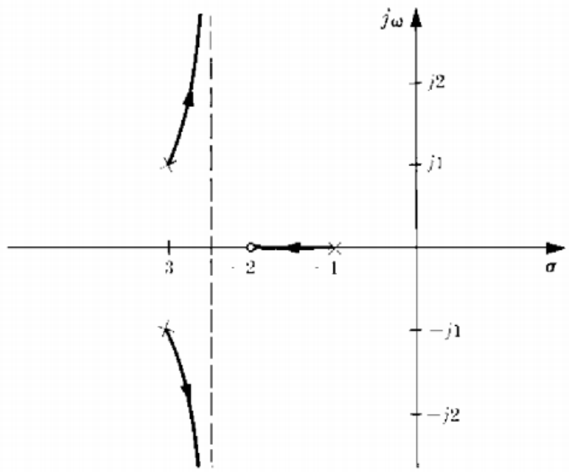

Fig. 13-30