1-01-f - Tutorial – Ruth Hurwitz Stability Criterion

ROUTH STABILITY CRITERION

5.9. Determine if the following characteristic equation represents a stable system:

$$s^3 + 4s^2 + 8s + 12 = 0$$

5.10. Determine if the following characteristic equation has any roots with positive real parts:

$$s^4 + s^3 - s - 1 = 0$$

5.11. The characteristic equation of a given system is

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

What restrictions must be placed upon the parameter K in order to insure that the system is stable?

5.12. Construct a Routh table and determine the number of roots with positive real parts for the equation

$$2s^3 + 4s^2 + 4s + 12 = 0$$

SOLUTION

ROUTH STABILITY CRITERION

5.9. Determine if the following characteristic equation represents a stable system:

$$s^3 + 4s^2 + 8s + 12 = 0$$

The Routh table for this system is

s^3	1	8
s^2	4	12
s^1	5	0
s^0	12	

Since there are no changes of sign in the first column, all the roots of the characteristic equation have negative real parts and the system is stable.

5.10. Determine if the following characteristic equation has any roots with positive real parts:

$$s^4 + s^3 - s - 1 = 0$$

Note that the coefficient of the s^2 term is zero. The Routh table for this equation is

s^4	1	0	−1
s^3	1	−1	0
s^2	1	−1	
s^1	0	0	
s^0	−1		

The coefficient for the s^0 row was obtained by replacing the 0 of the s^1 row by ϵ and then computing the coefficient of the s^0 row as

$$\frac{\epsilon(-1) - 0}{\epsilon} = -1$$

This procedure is necessary when a zero is obtained in the first column. Since there is one change of sign, the characteristic equation has one root with a positive real part. The presence of the zeros in the s^1 row indicates that the characteristic equation has two roots which satisfy the auxiliary equation formed from the s^2 row as follows: $s^2 - 1 = 0$. The roots of this equation are +1 and −1.

5.11. The characteristic equation of a given system is

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

What restrictions must be placed upon the parameter K in order to insure that the system is stable?

The Routh table for this system is

s^4	1	11	K
s^3	6	6	0
s^2	10	K	0
s^1	$\frac{60-6K}{10}$	0	
s^0	K		

For the system to be stable, the following restrictions must be placed upon the parameter K: $60 - 6K > 0$ or $K < 10$, and $K > 0$. Thus K must be greater than zero and less than 10.

5.12. Construct a Routh table and determine the number of roots with positive real parts for the equation

$$2s^3 + 4s^2 + 4s + 12 = 0$$

The Routh table for this equation is given below. Here the s^2 row was divided by 4 before the s^1 row was computed. The s^1 row was then divided by 2 before the s^0 row was computed.

s^3	2	4
s^2	1	3
s^1	-1	0
s^0	3	

Since there are two changes of sign in the first column of the Routh table, the equation above has two roots with positive real parts.