1-01-d – Tutorial - Solution

Question 1

8.6. Construct a signal flow graph for the simple resistance network given in Figure 8-13.

$$i_1 = \left(\frac{1}{R_1}\right)v_1 - \left(\frac{1}{R_1}\right)v_2 \qquad v_2 = R_3 i_1 - R_3 i_2 \qquad i_2 = \left(\frac{1}{R_2}\right)v_2 - \left(\frac{1}{R_2}\right)v_3 \qquad v_3 = R_4 i_2$$

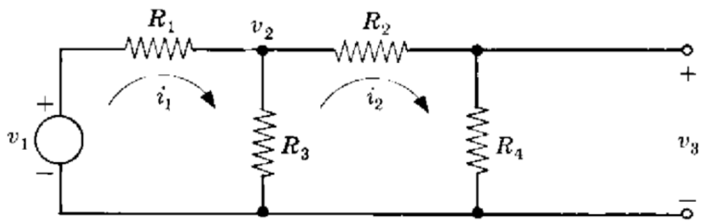

Fig. 8-13

SOLUTION

Laying out the five nodes in the same order with v_1 as an input node, and connecting the nodes with the appropriate branches, we get Fig. 8-14. If we wish to consider v_3 as an output node, we must add a unity gain

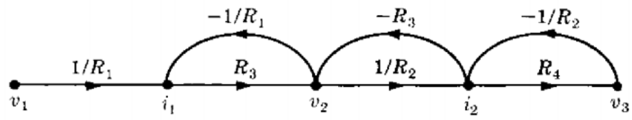

Fig. 8-14

branch and another node, yielding Fig. 8-15. No rearrangement of the nodes is necessary. We have one forward path and three feedback loops clearly in evidence.

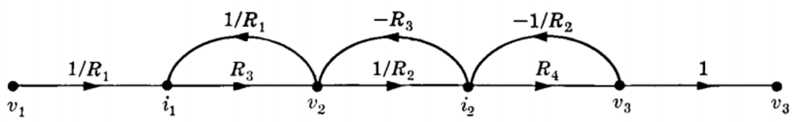

Fig. 8-15

Note that signal flow graph representations of equations are not unique. For example, the addition of a unity gain branch followed by a dummy node changes the graph, but not the equations it represents.

Question 2

EXAMPLE 8.8. The signal flow graph of the resistance network of Example 8.6 is shown in Fig. 8-17. Let us apply Equation (*8.2*) to this graph and determine the voltage gain $T = v_3/v_1$ of the resistance network.

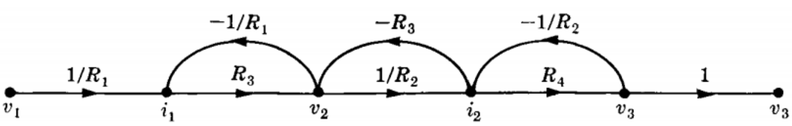

Fig. 8-17

SOLUTION

There is one forward path (Fig. 8-18). Hence the forward path gain is

$$P_1 = \frac{R_3 R_4}{R_1 R_2}$$

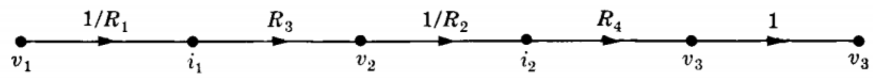

Fig. 8-18

There are three feedback loops (Fig. 8-19). Hence the loop gains are

$$P_{11} = -\frac{R_3}{R_1} \qquad P_{21} = -\frac{R_3}{R_2} \qquad P_{31} = -\frac{R_4}{R_2}$$

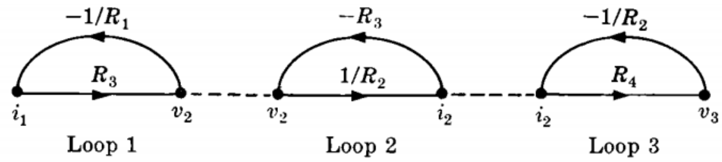

Fig. 8-19

There are two nontouching loops, loops 1 and 3. Hence

$$P_{12} = \text{gain product of the only two nontouching loops} = P_{11} \cdot P_{31} = \frac{R_3 R_4}{R_1 R_2}$$

There are no three loops that do not touch. Therefore

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12} = 1 + \frac{R_3}{R_1} + \frac{R_3}{R_2} + \frac{R_4}{R_2} + \frac{R_3 R_4}{R_1 R_2}$$
$$= \frac{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_3 R_4}{R_1 R_2}$$

Since all loops touch the forward path, $\Delta_1 = 1$. Finally,

$$\frac{v_3}{v_1} = \frac{P_1 \Delta_1}{\Delta} = \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_3 R_4}$$

Question 3

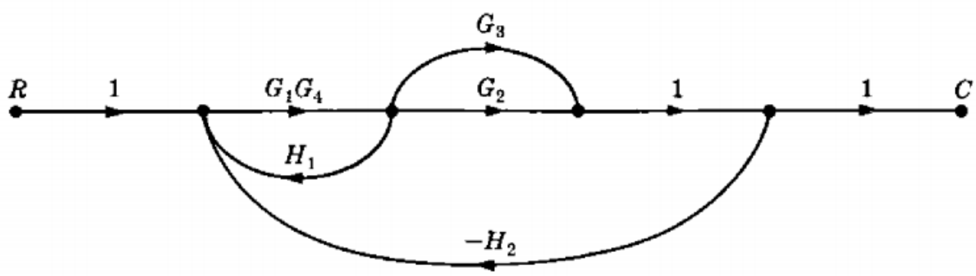

Fig. 8-26

SOLUTION

The signal flow graph is given in Fig. 8-26. There are two forward paths:

$$P_1 = G_1G_2G_4 \qquad P_2 = G_1G_3G_4$$

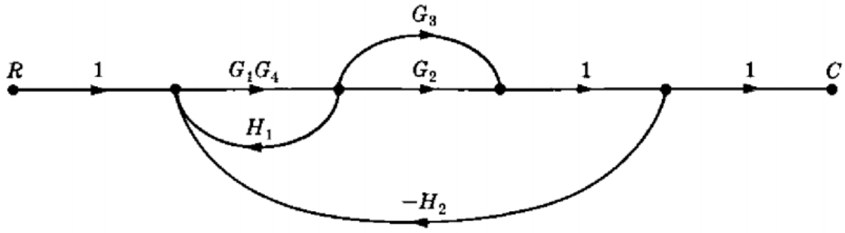

Fig. 8-26

There are three feedback loops:

$$P_{11} = G_1G_4H_1 \qquad P_{21} = -G_1G_2G_4H_2 \qquad P_{31} = -G_1G_3G_4H_2$$

There are no nontouching loops, and all loops touch both forward paths; then

$$\Delta_1 = 1 \qquad \Delta_2 = 1$$

Therefore the control ratio is

$$T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Question 4

8.7. Construct the signal flow graph for the following set of simultaneous equations:

$$x_2 = A_{21}x_1 + A_{23}x_3 \qquad x_3 = A_{31}x_1 + A_{32}x_2 + A_{33}x_3 \qquad x_4 = A_{42}x_2 + A_{43}x_3$$

SOLUTION

There are four variables: $x_1, \ldots, x_4$. Hence four nodes are required. Arranging them from left to right and connecting them with the appropriate branches, we obtain Fig. 8-39.

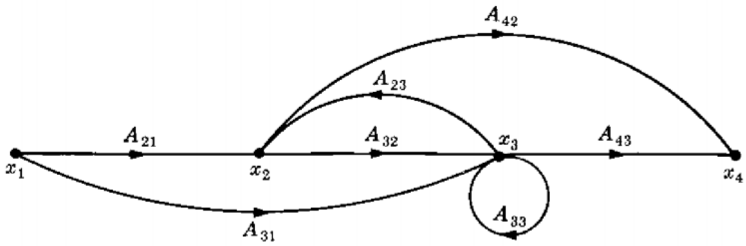

Fig. 8-39

A neater way to arrange this graph is shown in Fig. 8-40.

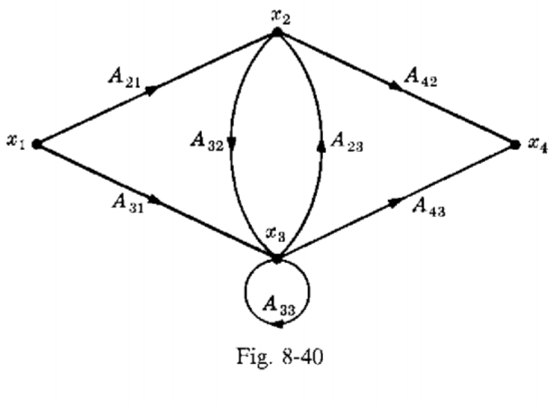

Fig. 8-40

Question 5

8.4. Consider the signal flow graph given in Fig. 8-34.

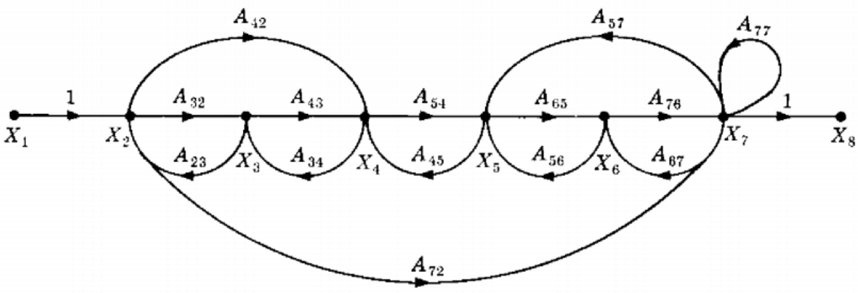

Fig. 8-34

Identify the (*a*) input node, (*b*) output node, (*c*) forward paths, (*d*) feedback paths, (*e*) self-loop. Determine the (*f*) loop gains of the feedback loops, (*g*) path gains of the forward paths.

SOLUTION

(*a*) X_1

(*b*) X_8

(*c*) X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8
X_1 to X_2 to X_7 to X_8
X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

(*d*) X_2 to X_3 to X_2; X_3 to X_4 to X_3; X_4 to X_5 to X_4; X_2 to X_4 to X_3 to X_2;
X_2 to X_7 to X_5 to X_4 to X_3 to X_2; X_5 to X_6 to X_5; X_6 to X_7 to X_6;
X_5 to X_6 to X_7 to X_5; X_7 to X_7; X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X

(*e*) X_7 to X_7

(*f*) $A_{32}A_{23}$; $A_{43}A_{34}$; $A_{54}A_{45}$; $A_{65}A_{56}$; $A_{76}A_{67}$; $A_{65}A_{76}A_{57}$; A_{77}; $A_{42}A_{34}A_{23}$;
$A_{72}A_{57}A_{45}A_{34}A_{23}$; $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$

(*g*) $A_{32}A_{43}A_{54}A_{65}A_{76}$; A_{72}; $A_{42}A_{54}A_{65}A_{76}$