Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 01-c

Transfer Function and System Block Diagrams

Contents

Reference:
Chapter 3, "Modern Control Engineering," K. Ogata, Prentice Hall
Feedback Control Systems – Schaum's Outline Series

Email: norbertcheung@szu.edu.cn
Web Site: http://norbert.idv.hk
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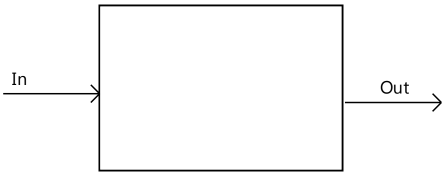
1. Linear and Nonlinear Systems

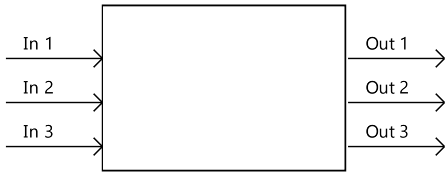
The *Mathematical Model* of a dynamic System is defined as a set of equations that represent the dynamics of the system accurately. Mathematical models may have different forms (e.g. differential equations, Laplace, State Space, etc.)

Once Mathematic Model of a system is obtained, various analytical and computer tools can be used for analysis and synthesis purposes.

For our case, we concentrate on studying *Single-Input-Single-Output*, *Linear*, *Time-Invariant Systems*.



SISO System (e.g. Air Conditioner)



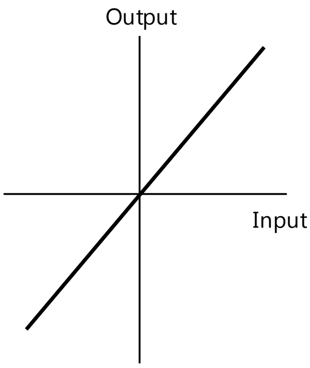
MIMO System (e.g. Air Craft Control)

Linear Systems obey the principle of superposition. The principle of superposition states that the response produced by the simultaneous applications of two different forcing functions is the sum of the two individual responses.

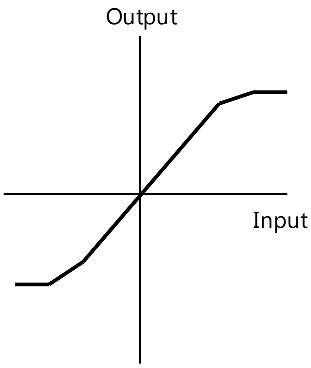
Nonlinear Systems does not obey the Law of Superposition. Examples are:

$$\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+x=A\sin\omega t$$

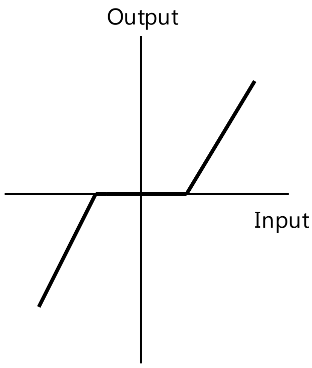
$$\frac{d^2x}{dt^2}+\left(x^2-1\right)\frac{dx}{dt}+x=0$$



Linear System



Saturation Nonlinearity



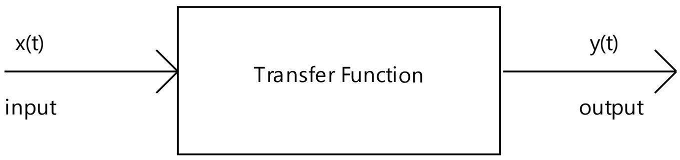
Dead Zone Nonlinearity

Linear Time-Invariant Systems have coefficients that are constants or functions only of the independent variable. Systems that are represented by differential equations whose coefficients are functions of time are called linear time-varying systems. An example is the space-craft control system, whose dynamic characteristics changes with time, because of the fuel consumption.

Linearization of nonlinear system involves a technique in which the control system is designed to have an operating point with a limited range, in which the characteristics inside this range is approximately linear.

<u>2. Transfer Function and Impulse Response Function</u>

A *Transfer Function* is used to specify the input output relationship of a component or system.



The linear time-invariant transfer function has the following form:

$$a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_{mx}$$

(n≥m)

where y is the output and x is the input.

Assuming all initial conditions are zero, we can take the Laplace Transform of both sides:

$$Transfer\_function = G(s) = \left.\frac{\zeta[output]}{\zeta[input]}\right|_{zero\_initial\_condition}$$

$$= \frac{Y(s)}{X(s)}$$

$$= \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n+1} + \cdots + a_{n-1}s + a_n}$$

If the highest power is n, the system is called nth order system

Transfer Function…..

1. Is a mathematical model expressed in differential equations that relates the output to the input.

2. Is the property of the system. It is independent of the input.

3. Does not provide the physical nature of the system. Therefore, two different physical systems can have the same transfer function.

4. Can be used to predict the output or response of the system. (Simulation)

5. Of an unknown system can be established experimentally by studying the inputs and outputs of a system. (System Identification)

How to obtain the Transfer function?

1. Write the differential equation for the system
2. Find its Laplace Transform, assuming all conditions are zero.
3. Find the ratio of the output against its input
4. This ratio is the transfer function

The Impulse Response Function

To find the output response of a system when it is subjected to a unit impulse response:

$$Y(s) = G(s) \times \text{impulse response}$$

In s-domain, the impulse response is unity, hence the equation can be simplified to:

$$Y(s) = G(s)$$

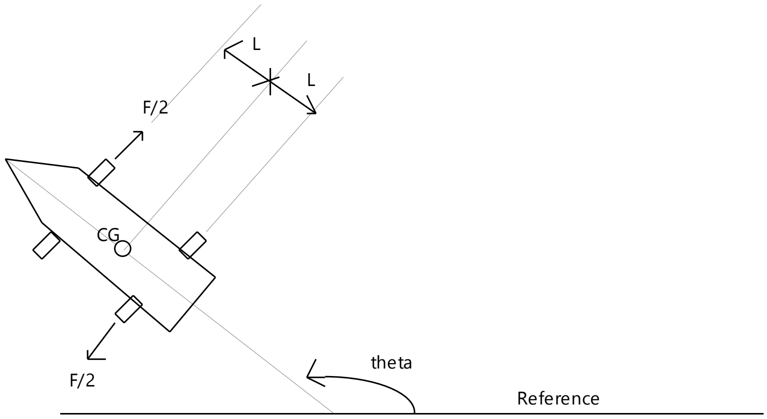
The output is actually equal to the transfer function. To find the response in time domain:

$$\zeta^{-1}[G(s)] = g(t)$$

g(t) can be treated as the transfer function in the time domain. It is also called the *impulse response function*. Experimentally, if we want to find the transfer function of a system, we can feed a unit impulse to the input, and measure the response from the output.

Example in Transfer Function

Consider a satellite control system shown in the figure below. Find its Transfer Function.



Solution

Thrust of each jet: F/2,

Therefore, Torque T= FL

Moment of Inertia = J

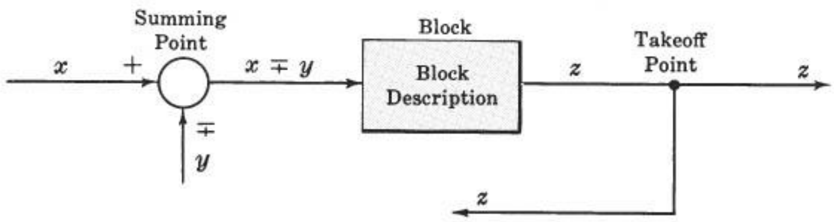
Therefore, Torque as a function of time can be written as: $J\dfrac{d^2\theta}{dt^2} = T$

Taking the Laplace transform of both sides: $Js^2\Theta(s) = T(s)$

The transfer function is output/input: $\dfrac{\Theta(s)}{T(s)} = \dfrac{1}{Js^2}$

3. Block Diagram of Feedback Control System

A Block Diagram is a pictorial representation of the function by each component, and of the flow of signals. It can display the system and signal flow more clearly.



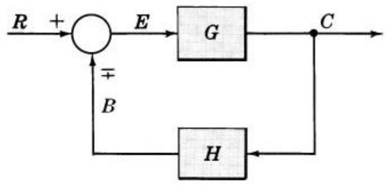
Block – Specifies the transfer function and the signal flow. The block contains at least two arrows; one input and one output.

Summing Point – Sums two or more signals together. The input can be positive or negative, an it is dictated by the input (+) or (-) sign.

Branch (takeoff) Point – A place in which the signal goes to two or more places.

Blocks in Cascade:

Feedback Control Block Diagram Summary:



E(s)	Error signal
C(s)	Output of the forward block
B(s)	Output of the feedback block
R(s)	Reference, or command signal
G(s)	Transfer function of the forward block
H(s)	Transfer function of the feedback block

Definition 7.1: $G \equiv$ direct transfer function $\equiv$ forward transfer function

Definition 7.2: $H \equiv$ feedback transfer function

Definition 7.3: $GH \equiv$ loop transfer function $\equiv$ open-loop transfer function

Definition 7.4: $C/R \equiv$ closed-loop transfer function $\equiv$ control ratio

Definition 7.5: $E/R \equiv$ actuating signal ratio $\equiv$ error ratio

Definition 7.6: $B/R \equiv$ primary feedback ratio

In the following equations, the (-) sign refers to a positive feedback system, and the (+) sign refers to a negative feedback system:

$$\frac{C}{R} = \frac{G}{1 \pm GH} \tag{7.3}$$

$$\frac{E}{R} = \frac{1}{1 \pm GH} \tag{7.4}$$

$$\frac{B}{R} = \frac{GH}{1 \pm GH} \tag{7.5}$$

The *Open Loop Transfer Function* is the ratio of the feedback signal to the actuating signal

$B(s)/E(s) = G(s)H(s)$

The ratio of the output to the actuating error is the *Feed Forward Transfer Function*:

$C(s)/E(s) = G(s)$

The Closed Loop Transfer Function can be obtained as follows:

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - B(s)$$

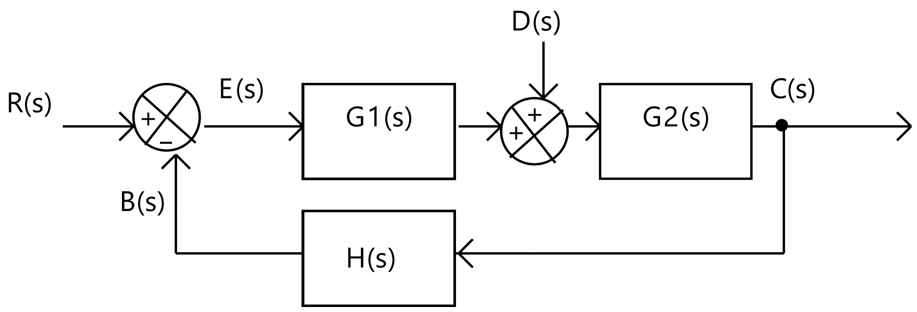
$$= R(s) - H(s)\,C(s)$$

Therefore:

$$C(s) = G(s)\,[R(s) - H(s)C(s)]$$

Or $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$ Output is $$C(s) = \frac{G(s)}{1+G(s)H(s)}R(s)$$

<u>For Close Loop System subject to disturbance</u>



Assume there is zero initial error, then the transfer function of D(s) to C(s) is:

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

If $G_1(s)G_2(s) >> 1$, then the function approaches zero

The input-output response of the system is

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

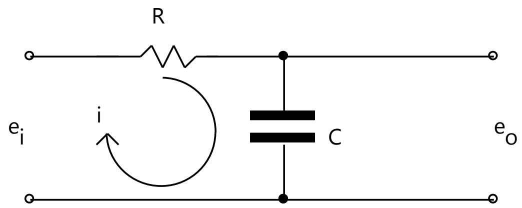
If $G_1(s)G_2(s) >> 1$, then the function approaches *1/H(s)*

This means that, for system with high feed forward gain:

- Disturbance will be eliminated
- Gain of system will approach unity if *H(s)* is 1
- Gain of system is independent of feed forward function
- Gain of system is dependent of feedback function

1-01-c <Transfer Function and System Block Diagrams>

<u>Procedure for drawing a block diagram</u> (with feedback), an example:

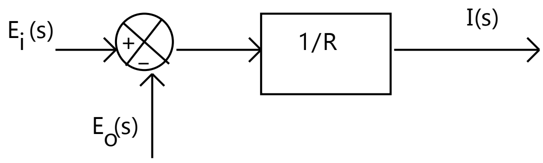


$$i = \frac{e_i - e_o}{R} \qquad \text{and} \qquad e_o = \frac{\int i dt}{C}$$

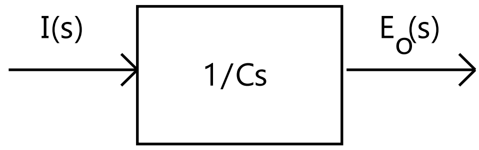
Take the Laplace Transform of the two equations:

$$I(s) = \frac{E_i(s) - E_o(s)}{R} \qquad \text{and} \qquad E_o(s) = \frac{I(s)}{sC}$$

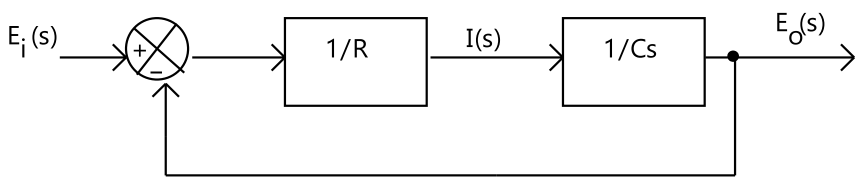
Transfer the first equation into block diagram



Transfer the second equation into block diagram

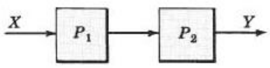
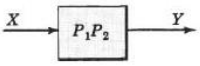
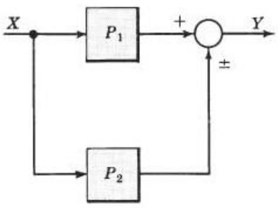
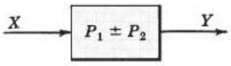
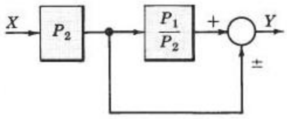
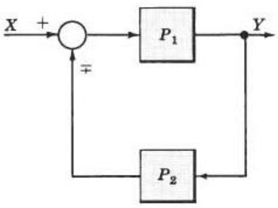
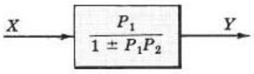
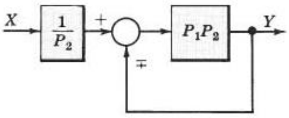
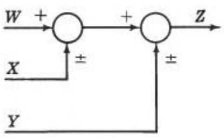
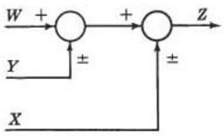
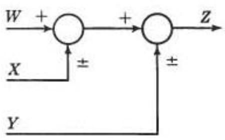
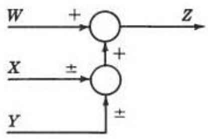
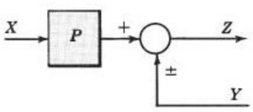
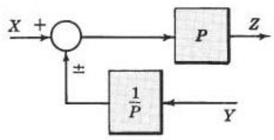
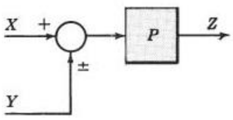
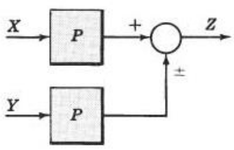


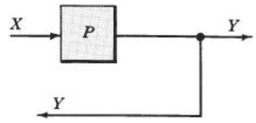
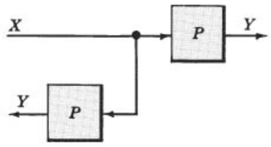
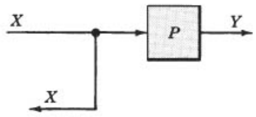
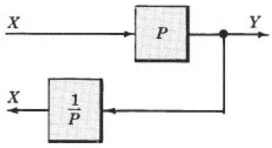
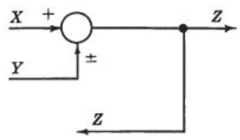
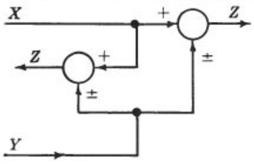
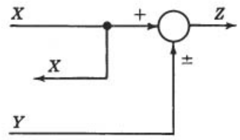
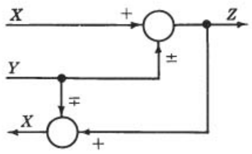
Then complete the whole picture



Done!

4. Block Diagram Reduction

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1X \pm P_2X$		
3	Removing a Block from a Forward Path	$Y = P_1X \pm P_2X$		
4	Eliminating a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
5	Removing a Block from a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
6a	Rearranging Summing Points	$Z = W \pm X \pm Y$		
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = PX \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P[X \pm Y]$		

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

VERY IMPORTANT

In simplifying the block diagram, the following rule must apply:

1. The product of the transfer function in the feed forward direction must remain the same.
2. The product of the transfer functions around the loop must remain the same.

Reduction of Complicated Block Diagrams

Step 1: Combine all cascade blocks using Transformation 1.

Step 2: Combine all parallel blocks using Transformation 2.

Step 3: Eliminate all minor feedback loops using Transformation 4.

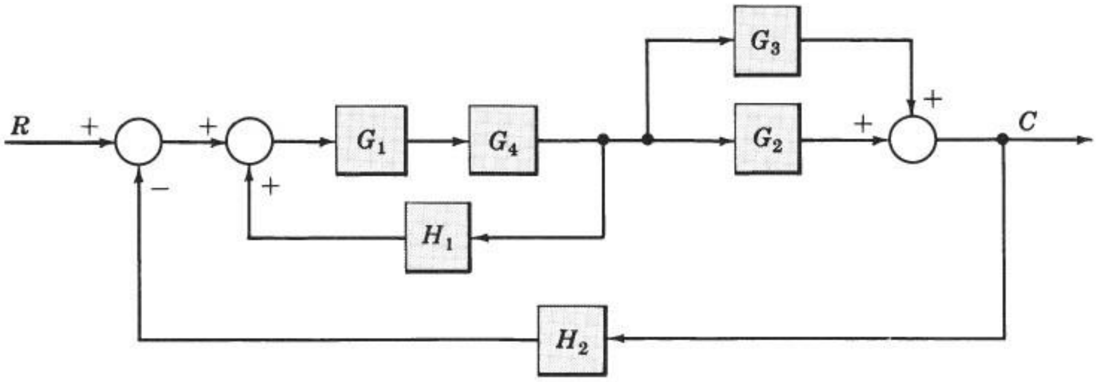
Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12.

Step 5: Repeat Steps-J to 4 until the canonical form has been achieved for a particular input.

Step 6: Repeat Steps 1 to 5 for each input, as required.

Transformations 3, 5, 6, 8, 9, and 11 are sometimes useful, and experience with the reduction technique will determine their application.

Example 1



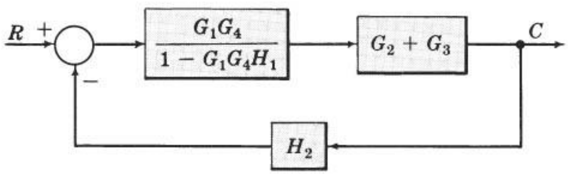
Step 1:

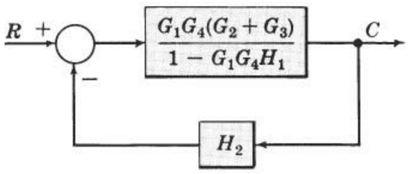
Step 2

Step 3

Step 4 Does not apply

Step 5

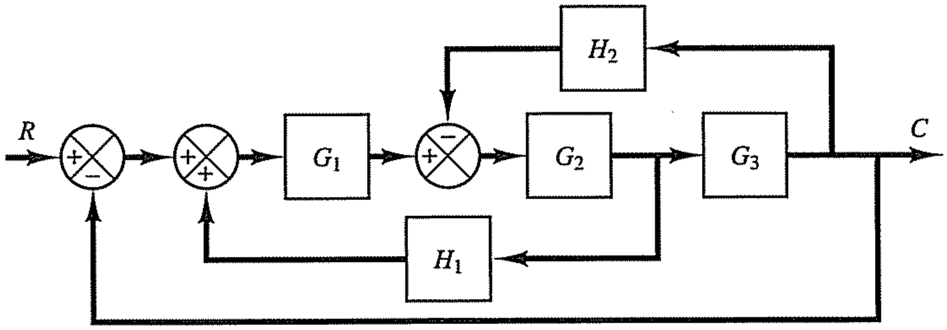




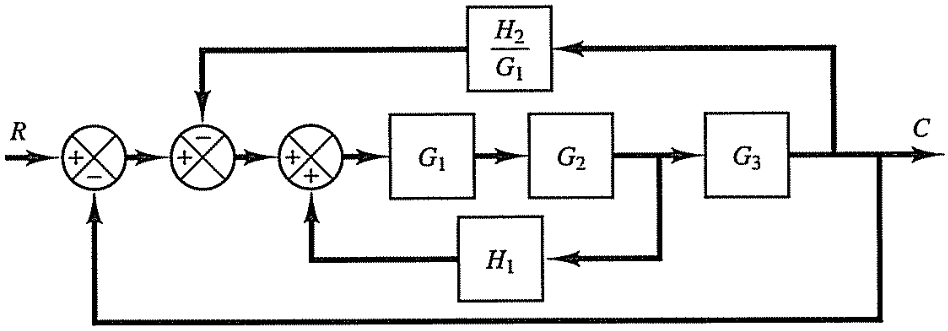
Step 6 Does not apply

Example 2

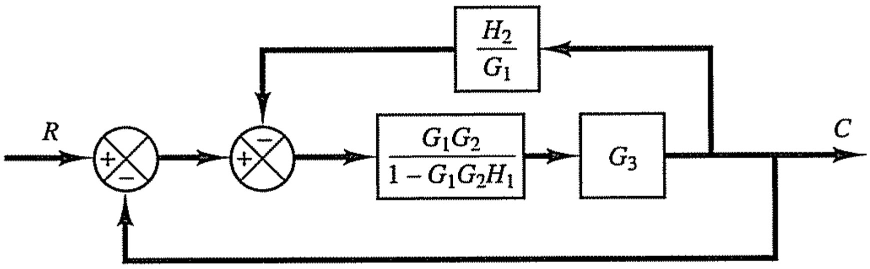
(a)



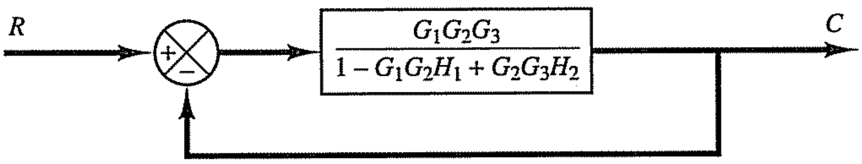
(b)



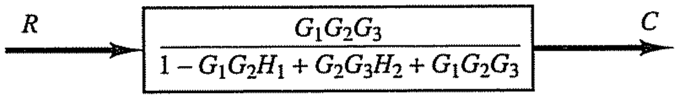
(c)



(d)



(e)



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Glossary – English/Chinese Translation

English	Chinese
Transfer Function	传递函数
System Block Diagram	系统框图
Linear System	线性系统
Non-Linear System	非线性系统
Block Diagram	方框图
Block Diagram Reduction	框图缩减
Mathematical Model	数学模型
Single Input Single Output	单输入单输出
Linear Time Invariant System	线性时不变系统
Law of Superposition	叠加定律
Saturation and Dead Zone	饱和度和死区
Impulse Response Function	脉冲响应功能
System Identification	系统识别
Moment of Inertia	转动惯量
Direct Transfer Function	直接传递功能
Feedback Transfer Function	反馈传递函数
Open Loop and Closed Loop	开环和闭环
Disturbance	外部干扰
Transformation	转型

Your Notes: