# 1-01-b Tutorial Solution

## Question 1

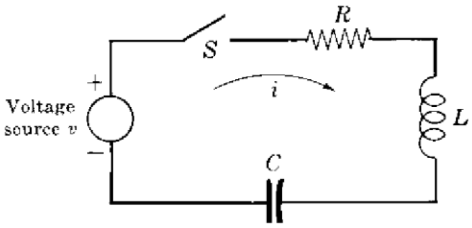


Fig. 3-6

By Kirchhoff's voltage law, the applied voltage $v(t)$ is equal to the sum of the voltage drops $v_R$, $v_L$, and $v_C$ across the resistor $R$, the inductor $L$, and the capacitor $C$, respectively. Thus

$$v = v_R + v_L + v_C = Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(\tau)\,d\tau$$

To eliminate the integral, both sides of the equation are differentiated with respect to time, resulting in the desired differential equation:

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv}{dt}$$

## Question 2

(a)

$$F(s) = \frac{s^2 + 2s + 2}{s^2 + 3s + 2} = \frac{s^2 + 2s + 2}{(s+1)(s+2)}$$

The partial fraction expansion of $F(s)$ is

$$F(s) = b_2 + \frac{c_{11}}{s+1} + \frac{c_{21}}{s+2}$$

The numerator coefficient of $s^2$ is $b_2 = 1$. The coefficients $c_{11}$ and $c_{21}$ are determined from Equation (*4.11b*) as

$$c_{11} = (s+1)F(s)\big|_{s=-1} = \left.\frac{s^2 + 2s + 2}{s+2}\right|_{s=-1} = 1$$

$$c_{21} = (s+2)F(s)\big|_{s=-2} = \left.\frac{s^2 + 2s + 2}{s+1}\right|_{s=-2} = -2$$

Hence $$F(s) = 1 + \frac{1}{s+1} - \frac{2}{s+2}$$

(b)

$$F(s) = \frac{1}{(s+1)^2(s+2)}$$

given by

$$\mathscr{L}^{-1}\left[\frac{1}{(s+1)^2(s+2)}\right] = \mathscr{L}^{-1}\left[-\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}\right]$$

$$= -\mathscr{L}^{-1}\left[\frac{1}{s+1}\right] + \mathscr{L}^{-1}\left[\frac{1}{(s+1)^2}\right] + \mathscr{L}^{-1}\left[\frac{1}{s+2}\right] = -e^{-t} + te^{-t} + e^{-2t}$$

Question 3

**EXAMPLE 4.5.** The Laplace transform of $(d/dt)(e^{-t})$ can be determined by application of Property 3. Since $\mathscr{L}[e^{-t}] = 1/(s+1)$ and $\lim_{t\to 0} e^{-t} = 1$, then

$$\mathscr{L}\left[\frac{d}{dt}(e^{-t})\right] = s\left(\frac{1}{s+1}\right) - 1 = \frac{-1}{s+1}$$

**EXAMPLE 4.6.** The Laplace transform of $\int_0^t e^{-\tau}\, d\tau$ can be determined by application of Property 4. Since $\mathscr{L}[e^{-t}] = 1/(s+1)$, then

$$\mathscr{L}\left[\int_0^t e^{-\tau}\, d\tau\right] = \frac{1}{s}\left(\frac{1}{s+1}\right) = \frac{1}{s(s+1)}$$

Question 4

**EXAMPLE 4.7.** The Laplace transform of $e^{-3t}$ is $\mathscr{L}[e^{-3t}] = 1/(s+3)$. The initial value of $e^{-3t}$ can be determined by the Initial Value Theorem as

$$\lim_{t\to 0} e^{-3t} = \lim_{s\to\infty} s\left(\frac{1}{s+3}\right) = 1$$

**EXAMPLE 4.8.** The Laplace transform of the function $(1 - e^{-t})$ is $1/s(s+1)$. The final value of this function can be determined from the Final Value Theorem as

$$\lim_{t\to\infty} (1 - e^{-t}) = \lim_{s\to 0} \frac{s}{s(s+1)} = 1$$

Question 5

Find the solution $x(t)$ of the differential equation

$$\ddot{x} + 3\dot{x} + 2x = 0, \qquad x(0) = a, \qquad \dot{x}(0) = b$$

where $a$ and $b$ are constants.

By writing the Laplace transform of $x(t)$ as $X(s)$ or

$$\mathscr{L}[x(t)] = X(s)$$

we obtain

$$\mathscr{L}[\dot{x}] = sX(s) - x(0)$$
$$\mathscr{L}[\ddot{x}] = s^2X(s) - sx(0) - \dot{x}(0)$$

And so the given differential equation becomes

$$[s^2X(s) - sx(0) - \dot{x}(0)] + 3[sX(s) - x(0)] + 2X(s) = 0$$

By substituting the given initial conditions into this last equation, we obtain

$$[s^2X(s) - as - b] + 3[sX(s) - a] + 2X(s) = 0$$

or

$$(s^2 + 3s + 2)X(s) = as + b + 3a$$

Solving for $X(s)$, we have

$$X(s) = \frac{as + b + 3a}{s^2 + 3s + 2} = \frac{as + b + 3a}{(s+1)(s+2)} = \frac{2a + b}{s+1} - \frac{a+b}{s+2}$$

The inverse Laplace transform of $X(s)$ gives

$$x(t) = \mathscr{L}^{-1}[X(s)] = \mathscr{L}^{-1}\left[\frac{2a+b}{s+1}\right] - \mathscr{L}^{-1}\left[\frac{a+b}{s+2}\right]$$
$$= (2a + b)e^{-t} - (a + b)e^{-2t}, \qquad \text{for } t \geq 0$$

which is the solution of the given differential equation. Notice that the initial conditions $a$ and $b$ appear in the solution. Thus $x(t)$ has no undetermined constants.