2004 – EE4008 – Test 2 (25% for each question)

Question 1

The PID algorithm can be summarized as follows:

$$u_k = u_{k-1} + \left(K_p + \frac{K_i T}{2} + \frac{K_d}{T}\right)e_k + \left(\frac{K_i T}{2} - K_p - \frac{2K_d}{T}\right)e_{k-1} + \frac{K_d}{T}e_{k-2}$$

Use a flow chart to explain how you would implement this algorithm inside the digital controller.

Question 2

A 2nd order Butterworth filter has the following characteristics:

Order	$H_B(s)$
2	$\frac{1}{s^2 + 1.414s + 1}$

(a) Design a high pass filter *H(s)*, with a cut off frequency of $\omega_0 = \sqrt{2}$.

(b) Convert into digital form *H(z)*, using bilinear transformation integration. Assume the sampling time is T.

Question 3

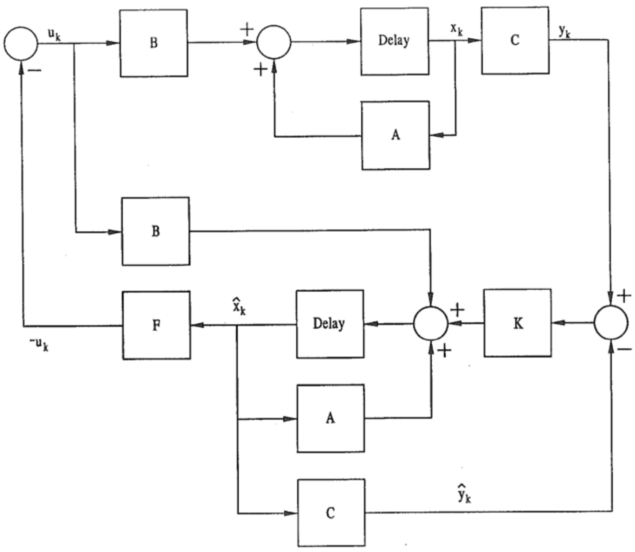
For the function: $F(z) = \dfrac{2z^2 + 3z}{z^2 + 4z - 5}$

(a) Find the A, B, C matrix of the controllable canonical form state space representation.

(b) Draw the flow diagram of the above function.

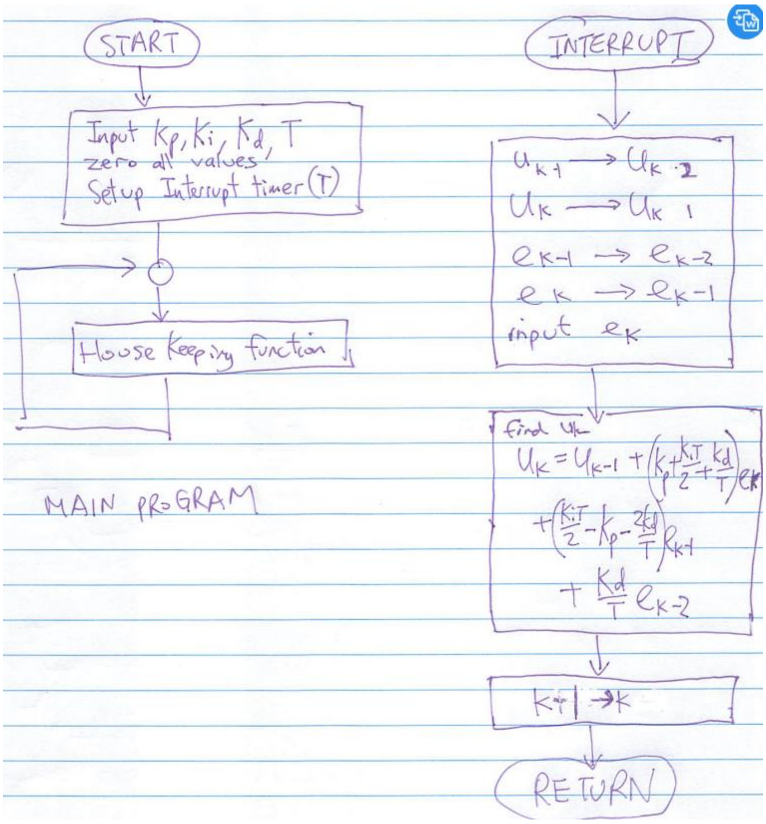
Question 4

With reference to the diagram below, describe (briefly and concisely) its operation principle.



SOLUTION

Question 1



Question 2

(a) $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$ for high pass filter $s \longrightarrow \frac{\omega_0}{s} = \frac{\sqrt{2}}{s}$

$$\therefore\ H(s) = \frac{1}{\left(\frac{\sqrt{2}}{s}\right)^2 + \sqrt{2}\left(\frac{\sqrt{2}}{s}\right) + 1} = \frac{s^2}{s^2+2s+2}$$

(b) for bilinear transform $s = \frac{2}{T}\cdot\frac{(z-1)}{(z+1)}$

$$H(z) = \frac{\left(\frac{2}{T}\cdot\frac{z-1}{z+1}\right)^2}{\left(\frac{2}{T}\cdot\frac{z-1}{z+1}\right)^2 + 2\cdot\frac{2}{T}\left(\frac{z-1}{z+1}\right) + 2}$$

note that : T is the sampling time and if is not defined in the question

(T is NOT $\frac{1}{\omega_0}$)

Question 3

$$F(z) = \frac{2z^2+3z}{z^2+4z-5} = \frac{-5z+10}{z^2+4z-5} + 2$$

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = -4x_2(k) + 5x_1(k) + u(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

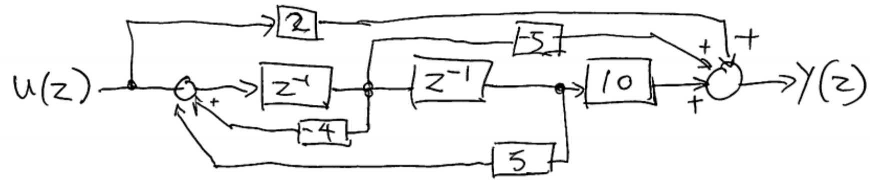
$$y(k) = \begin{bmatrix} 10 & -5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + 2u_k$$

Therefore

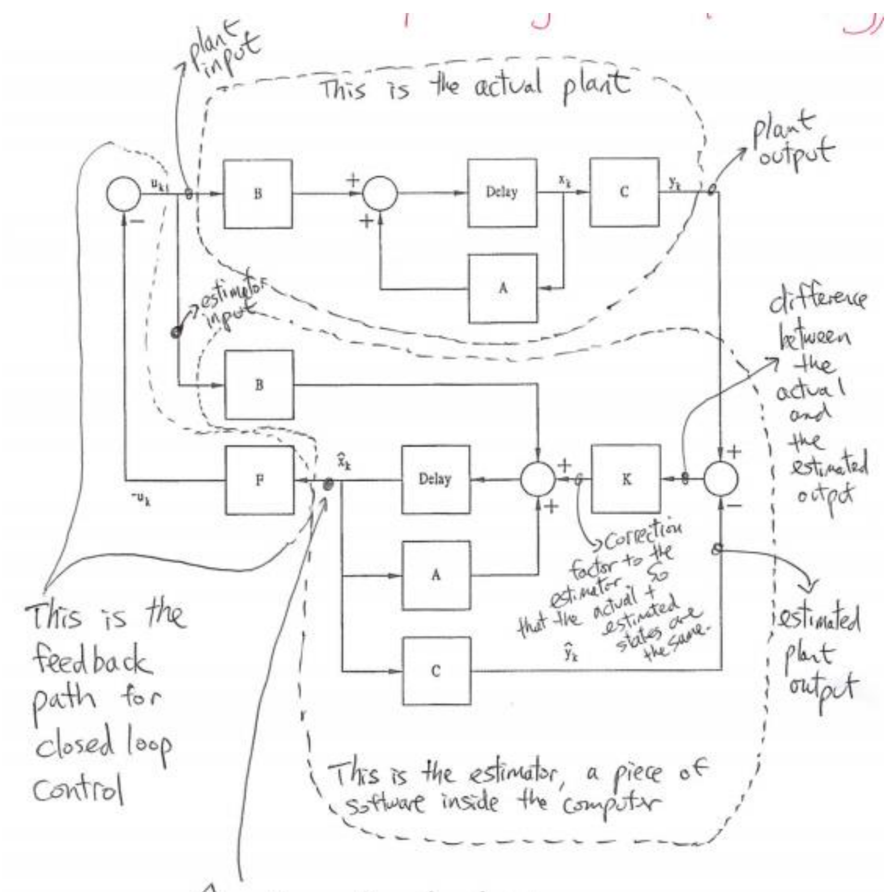
$A = \begin{bmatrix} 0 & 1 \\ 5 & -4 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 10, & -5 \end{bmatrix}$



Question 4



$\hat{x}_k$: these are the internal output estimated states of the estimator. It is used for closed loop feedback control of the actual plant.