Review on Calculus

Partial Fraction Decomposition

- In control systems analysis, *F*(*s*), usually occurs in the form $F(s) = \frac{A(s)}{B(s)}$, where *A*(*s*) and *B*(*s*) are polynomials

For example, $\frac{2s+5}{s^2+3s+2} = \frac{3}{s+1} - \frac{1}{s+2}$

- There are 4 types of partial fraction decomposition

(1) Non-repeated linear factors in denominator

$$\frac{F(s)}{(s+a)(s+b)(s+c)} = \frac{A}{s+a} + \frac{B}{s+b} + \frac{C}{s+c}$$

(2) Repeated linear factor in denominator

$$\frac{F(s)}{(s+a)^n} = \frac{A}{(s+a)^n} + \cdots + \frac{X}{(s+a)^2} + \frac{Y}{s+a}$$

Review on Calculus

Partial Fraction Decomposition

(3) Non-repeated quadratic factors in denominator

$$\frac{F(s)}{(s^2+as+b)(s^2+cs+d)} = \frac{As+B}{s^2+as+b} + \frac{Cs+D}{s^2+cs+d}$$

(4) Repeated quadratic factor in denominator

$$\frac{F(s)}{(s^2+as+b)^n} = \frac{As+B}{(s^2+as+b)^n} + \cdots + \frac{Ws+X}{(s^2+as+b)^2} + \frac{Ys+Z}{s^2+as+b}$$

Example 2

Find the partial fraction of

$$\frac{1}{(s^2-9)}$$

$$s = \frac{0 \pm \sqrt{0^2 - 4(1)(-9)}}{2(1)} = \pm 3$$

Answer:

Write

$$\frac{1}{s^2-9} = \frac{A}{s+3} + \frac{B}{s-3} = \frac{A(s-3)+B(s+3)}{s^2-9}$$

So that

$$A(s-3)+B(s+3) = 1$$

Put $s = 3$, and $s = -3$ respectively on both sides of the above equality. We have,

$$A(3-3)+B(3+3) = 1 \Rightarrow B = \frac{1}{6}$$

$$A(-3-3)+B(-3+3) = 1 \Rightarrow A = -\frac{1}{6}$$

$$\therefore \frac{1}{s^2-9} = -\frac{1}{6}\frac{1}{s+3} + \frac{1}{6}\frac{1}{s-3}$$

Example 3

Find the partial fraction of $\frac{1}{s(s+2)}$.

Answer:

Write $\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$ $= \frac{A(s+2)+Bs}{s(s+2)}$

$A(s+2)+Bs = 1$

$s = -2, \quad -2B = 1 \quad \Rightarrow B = -\frac{1}{2}$

$s = 0, \quad 2A = 1 \quad \Rightarrow A = \frac{1}{2}$

$\therefore \frac{1}{s(s+2)} = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2}$