DEPARTMENT OF ELECTRICAL ENGINEERING

SOLUTION & MARKING SCHEME

(Semester 2, 2022/23)

SUBJECT (Code & Title)	EE4008A Applied Digital Control

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SUBJECT MODERATOR	PT Lau

QUESTION NO.	SOLUTION	MARKS
PART A		
Q1 (a)	Micro-controller (embedded processor), features: • All interface on a chip • Less computing power • Low cost and high volume applications Digital Signal Processor (DSP), features: • Highly parallel architecture for signal processing • Include multiplier and barrel shifter hardware • May include high speed interface for parallel processing	5
Q1 (b)	Why use digital instead of analogue? • More stable performance, environmentally robust • Lower component cost, less setup and adjustment effort • Software Programmable • Can easily include more intelligent control functions • More space saving. • Easy interface with other systems	5

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<tr><th>QUESTION NO.</th><th>SOLUTION</th><th>MARKS</th></tr>
<tr><td>Q2 (a)</td><td>

Find the inverse sequence for the following function:

$$F(z) = \frac{z^2 + z}{z^2 - 3z + 4}$$

Multiplication by z^{-2} in numerator and denominator gives

$$F(z) = \frac{1 + z^{-1}}{1 - 3z^{-1} + 4z^{-2}}$$

Now carry out formal long division to yield

$$\begin{array}{r|l} & 1 + 4z^{-1} + 8z^{-2} \\ \hline 1 - 3z^{-1} + 4z^{-2} & 1 + z^{-1} \\ & \underline{1 - 3z^{-1} + 4z^{-2}} \\ & \quad 4z^{-1} - 4z^{-2} \\ & \quad \underline{4z^{-1} - 12z^{-2} + 16z^{-3}} \\ & \qquad 8z^{-2} - 16z^{-3} \\ & \qquad \underline{8z^{-2} - 24z^{-3} + 32z^{-4}} \\ & \qquad\qquad 8z^{-3} - 32z^{-4} \\ & \qquad\qquad \vdots \end{array}$$

Now upon examination of the coefficients of the infinite series answer, the sequence is

$$f(0) = 1$$
$$f(1) = 4$$
$$f(2) = 8$$

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<tr><td>Q2 (b)</td><td>

$$F(z) = \frac{z^2 + z}{(z - 0.6)(z - 0.8)(z - 1)}$$

Find the partial fraction expansion and invert the resulting transform. The expansion will be of the form

$$F(z) = \frac{A_1 z}{z - 0.6} + \frac{A_2 z}{z - 0.8} + \frac{A_3 z}{z - 1}$$

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$$A_1 = \left.\frac{z+1}{(z-0.8)(z-1)}\right|_{z=0.6} = \frac{1.6}{(-0.2)(-0.4)} = 20$$

$$A_2 = \left.\frac{z+1}{(z-0.6)(z-1)}\right|_{z=0.8} = \frac{1.8}{(0.2)(-0.2)} = -45$$

and

$$A_3 = \left.\frac{z+1}{(z-0.6)(z-0.8)}\right|_{z=1} = \frac{2}{(0.4)(0.2)} = 25$$

So upon inversion of the transform,

$$f(k) = 20(0.6)^k - 45(0.8)^k + 25$$

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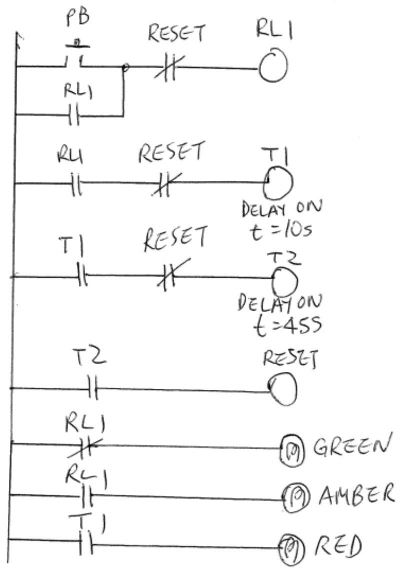
<tr><td>Q3</td><td>



Add explanations

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<tr><td>Q4</td><td>

$$H(z) = \frac{\sum_{n=0}^{M} b(n) z^{-n}}{1 + \sum_{n=1}^{N} a(n) z^{-n}}$$

$$= H_1(z) H_2(z) = \left[\frac{1}{1 + \sum_{n=1}^{N} a(n) z^{-n}}\right]\left[\sum_{n=0}^{M} b(n) z^{-n}\right]$$

H2(z) = $b_0 + b(1)z^{-1} + b(2)z^{-2} + b(3)z^{-3}$

$$H_1(z) = \frac{1}{1 + a(1)z^{-1} + a(2)z^{-2} + a(3)z^{-3}}$$

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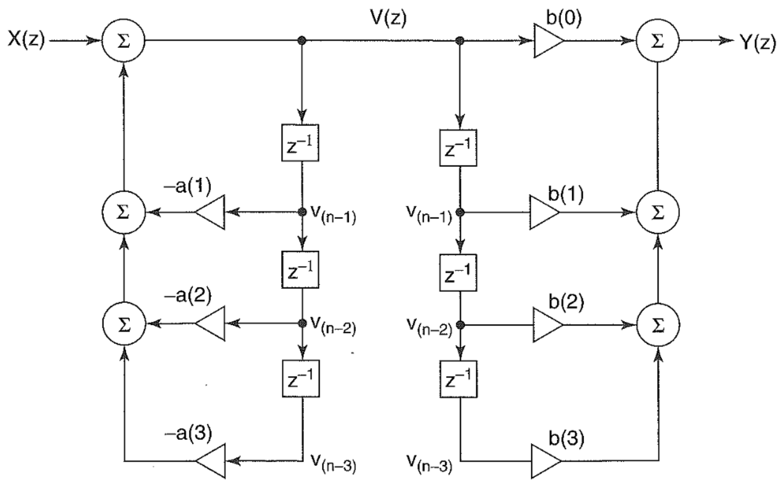

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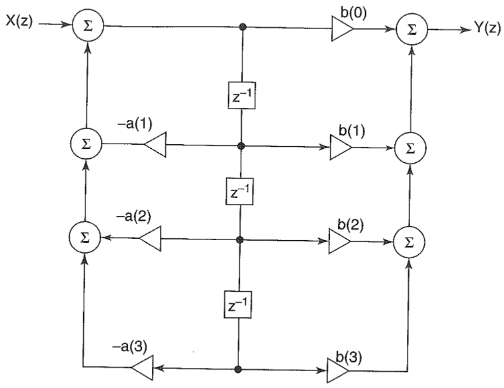
Further simplify….



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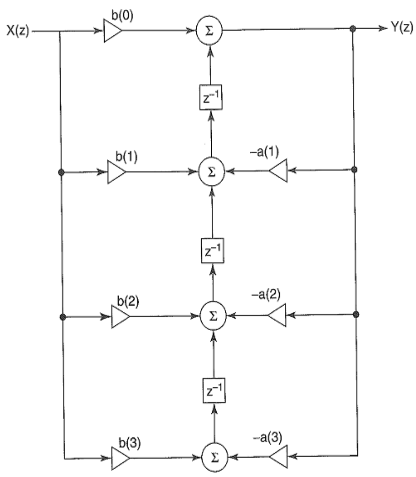

Minimum hardware achieved!
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<tr><td>PART B

Q1</td><td>
Q1:

$x(0) = \lim_{z\to\infty} X(z) = \lim_{z\to\infty} \frac{6}{1-z^{-2}} = 6$ 5'

$x(\infty) = \lim_{z\to 1} (1-z^{-1}) \frac{6}{1-z^{-2}} = \lim_{z\to 1} \frac{6}{1+z^{-1}} = \frac{6}{1+1} = 3$ 5'
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QUESTION NO.	SOLUTION	MARKS
2	Q2: A: ① $G_{ol}(z) = G_c(z)\,(1-z^{-1})\cdot \mathcal{Z}\left\{\frac{G_p(s)}{s}\right\}$ $= G_c(z)\cdot(1-z^{-1})\cdot\mathcal{Z}\left(\frac{1}{s(s+1)}\right)$ 5' $= \frac{z-e^{-1}}{(z-2)(z-1)}\cdot\frac{(1-z^{-1})z}{z-e^{-1}} = \frac{1}{z-2}$ $G_{cl}(z) = \frac{1}{z-1} \longrightarrow$ Marginally stable 5'	10
3	Q3: $G(s) = \frac{2}{s^2+2s+2} \overset{z}{=} \frac{2}{\left(\frac{z-1}{z}\right)^2 + 2\left(\frac{z-1}{z}\right)+2}$ $= \frac{2}{\frac{z^2+1-2z}{z^2}+\frac{2z-2}{z}+2} = \frac{2z^2}{z^2+1-2z+2z^2-2z+2z^2} = \frac{2z^2}{5z^2-4z+1}$ 5' $\frac{Y(z)}{X(z)} = \frac{2z^2}{5z^2-4z+1} = \frac{2}{5-4z^{-1}+z^{-2}}$ $\downarrow$ $5Y(z)-4z^{-1}Y(z)+z^{-2}Y(z) = 2X(z)$ 5' $\downarrow k$ $5y(k)-4y(k-1)+y(k-2) = 2x(k)$ 5'	15

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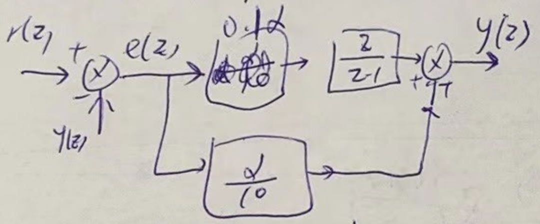
Q4: $G_{loop}(s) = \frac{\alpha}{s} \cdot \frac{1}{G_p(s)} = G_c(s)$

$G_c(s) = \frac{\alpha}{s} \cdot \frac{s+10}{10} = \frac{\alpha}{s} + \frac{\alpha}{10}$ 5'

$k_p = \frac{\alpha}{10}$ $k_I = \alpha$



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Declare kI. kP. Y(z)

$0.1\alpha \sum_{k=0}^{n} e(k) = kI$;

$\frac{\alpha}{10} \cdot e(k) = kp$

$y(z) = kp + kI$;

5'

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