## 3. Analysis of second order system

Consider a second order system

$$\frac{C(s)}{R(s)} = \frac{kb}{s^2 + as + b}$$

The denominator is quadratic in $s$.
System poles may be either real or complex.

System poles are complex if $a2 - 4b < 0$ and real if $a2 - 4b \geq 0$.

Rewrite the transfer function as:

$$\frac{C(s)}{R(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

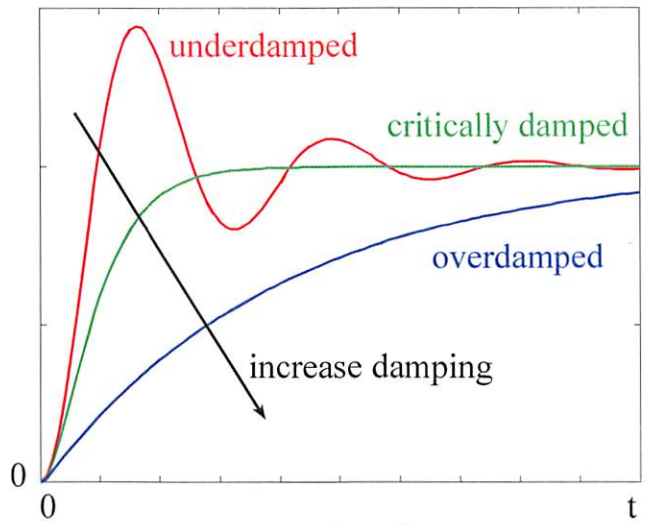
where $\omega_n = \sqrt{b}$ = undamped natural frequency

$$\zeta = \frac{a}{2\omega_n} = \text{damping ratio}$$

System poles will be complex only if:

$$0 < \zeta < 1$$

1. Underdamped case $(0 < \zeta < 1)$
2. Critically damped case $(\zeta = 1)$
3. Overdamped case $(\zeta > 1)$



**For the under damped case (0<ζ<1)**

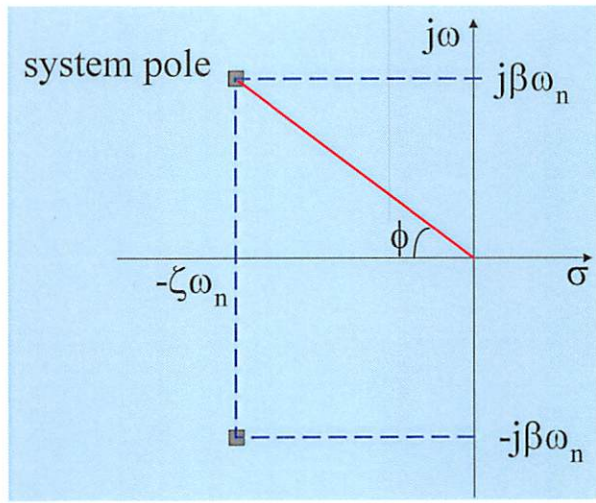
Characteristic equation: $\Delta(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

System poles : $s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n$

Define $\phi = \cos^{-1}\zeta$

and $\beta = \sin\phi$

$= \sqrt{1-\zeta^2}$




The unit step response is:

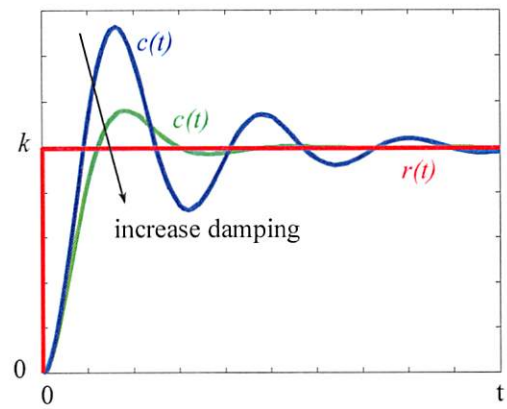
$$C(s) = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \bullet \frac{1}{s}$$

$$= \frac{k}{s} - \frac{k(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + (\beta\omega_n)^2} - \frac{k\zeta\omega_n}{(s+\zeta\omega_n)^2 + (\beta\omega_n)^2}$$

Taking the Laplace inverse transform:

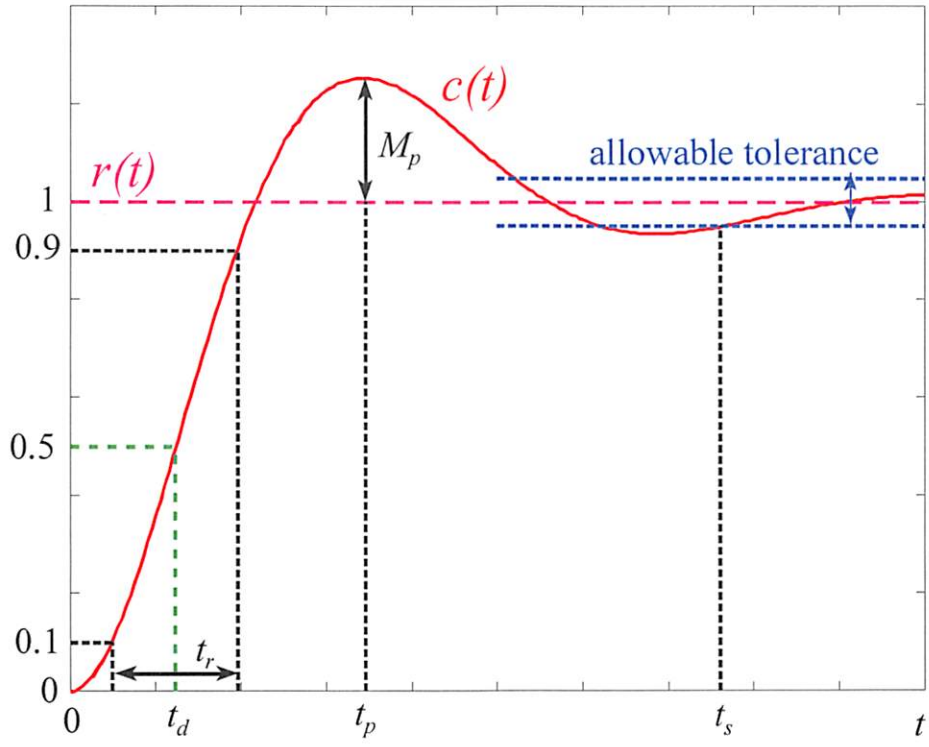
$$c(t) = k - ke^{-\zeta\omega_n t}\cos\beta\omega_n t - \frac{k\zeta e^{-\zeta\omega_n t}}{\beta}\sin\beta\omega_n t$$

$$= k - \frac{ke^{-\zeta\omega_n t}}{\beta}\sin(\beta\omega_n t + \phi)$$



## 4. Transient Response

Definition of transient response specifications




1. Delay time *td* : Time required for the response to reach half the final value.
2. Rise time *tr* : Time required to rise from 10% to 90% (overdamped) and 0 to 100% (underdamped) of its final value.
3. Peak time *tp* : Time required to reach the first peak of the overshoot.
4. Maximum overshoot *Mp* : Occur at the peak time *tp* .
5. Settling time *ts* : Time required to reach and stay within a range about the final value of size specified by absolute percentage of final value. (usually 5% or 2%)