

# Integral sliding mode control and its application on Active suspension system

J.K. Lin, K.W.E. Cheng, Z. Zhang, N.C. Cheung, X.D. Xue, M.K. Wong, D.H. Wang, Y.J. Bao, Jones Chan, John Lam

Department of Electrical Engineering, The Hong Kong Polytechnic University, Hong Kong  
E-mail: 09900854r@polyu.edu.hk

**Abstract**—Active suspension system is a hot issue in the recent research of automotive industry, which is to generate active force to suppress the variation of the vehicle and improve safety and comfort for the passengers. By observing the dynamics of the vehicle, the controller output forces through linear actuators. Integral sliding mode control is applied here to calculate the output forces and then reject the disturbance. In this paper, an accurate nonlinear active suspension system model is developed first; decoupling of the sprung mass subsystem is investigated hereafter to isolate the dependence of the variables, then integral sliding mode control is used to reject the disturbance and reach the referenced surface for each subsystem. PID control, as a conventional control method, is compared here with the integral sliding mode control. The simulation results show the effectiveness of the proposed control method.

**Keywords**—Active suspension system, Nonlinear model, Integral sliding mode control

## I. INTRODUCTION

Active suspension system has been focused by academic research and industrial application, which is supposed to eliminate or minimize vibrations greatly which ranged from 0.5 to 80 Hz [1], then isolate the vehicle body from road irregularities.

The active suspension system is a complex nonlinear system. The electrical active suspension system with linear actuators is proposed with the advantages of less mechanical connection and flexible control. Martins et al. (2006) proposed the potential of electrical active suspension system, and specified the “reduced comfort boundary” with one-third active force of the weight of the sprung mass [2]. Ikenaga et al (2000) has simplified the complicated full-car active suspension system into a linear model with filtered feedback control scheme under small vibration of the sprung mass.[3] An accurate full car ride model using model reducing techniques is proposed, and linearization of a full car MBD (multi-body dynamics) model is developed to obtain a large-order vehicle model. [4] However, linearization of the full vehicle is only effective based on the assumption that the vertical vibration is restricted within a small range, which is hard to satisfy under moderate road irregularities. An accurate nonlinear full car ride model is necessary to develop for accurate control of the actual car suspension system. A nonlinear active suspension system with four electro-hydraulic actuators is proposed by Chamseddine (2006) to illustrate the dynamic performances of the hydraulic active suspension system. [5]

Various control methods for active suspension system with electromagnetic actuator have been investigated. Multi-

objective optimal control strategy is pursued for finding feedback control laws via mixed  $H_2/H_\infty$  control [6]. A simple and effective linear model following control method is used to reduce the calculation burden [7]. For the purpose of fault diagnosis and tolerance, sliding mode controller (SMC) had been developed by Yagiz (2000) for a nonlinear vehicle model [8]; a modified SMC was designed for a linear full vehicle active suspension system with partly knowledge of the system states [9]. For the conventional SMC method, the desired dynamic can be obtained only when the sliding mode occurs. A novel type PID type sliding mode control, so-called integral sliding mode control (ISMC) is introduced by Utkin [10], in which the sliding mode occurs at the initial instant and therefore the robustness of the system can be guaranteed during the entire process.

In this paper, an accurate nonlinear model is proposed in Section 2, to describe the dynamic response of the active suspension system because the dynamic of the vertical vibration of the car body is beyond the assumption of the linear model. A brief introduction of integral sliding mode control method is presented in Section 3, and its application on active suspension system is developed to reject road irregularities and parameter variations. Simulation results against road irregularities and parameter variations are shown in Section 4, to verify the proposed ISMC controller. In last section, the conclusions are given.

## II. NONLINEAR MODEL OF ACTIVE SUSPENSION SYSTEM

The nonlinear model of the active suspension system is a 7-DOF system. Applying the principle of force and torque balance, with the consideration of the gravity of the sprung mass and unsprung mass and the fact of rigid body, the model is summarized as follow:

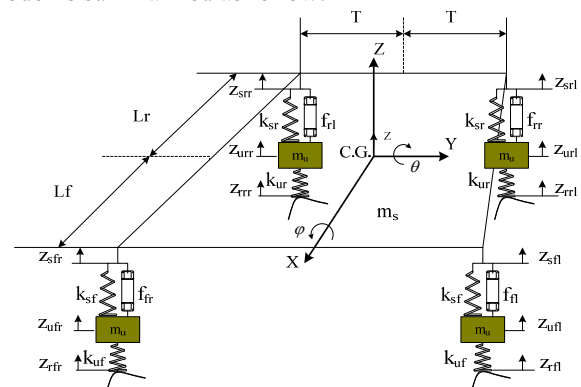


Fig.1: Model of the active suspension system

$$\begin{aligned}
 M_s \ddot{z} &= -(2K_{sf} + 2K_{sr})z - (2K_{sf}L_f - 2K_{sr}L_r)\sin\theta + K_{sf}(z_{uf} + z_{ur}) + K_{sr}(z_{ur} + z_{ur}) \\
 &\quad - M_s g + \cos\theta \cos\varphi (F_{fl} + F_{fr} + F_{rl} + F_{rr}) \\
 I_p \ddot{\theta} &= -(2K_{sf}L_f - 2K_{sr}L_r)\cos\theta z - (2K_{sf}L_f^2 + 2K_{sr}L_r^2)\cos\theta \sin\theta + K_{sf}L_f \cos\theta (z_{uf} + z_{ur}) \\
 &\quad - K_{sr}L_r \cos\theta (z_{ur} + z_{ur}) + L_f \cos^2\theta \cos\varphi (F_{fl} + F_{fr}) - L_r \cos^2\theta \cos\varphi (F_{rl} + F_{rr}) \\
 I_r \ddot{\varphi} &= -(2K_{sf} + 2K_{sr})T^2 \cos\varphi \sin\varphi + K_{sf}T \cos\varphi (z_{uf} - z_{ur}) + K_{sr}T \cos\varphi (z_{ur} - z_{ur}) \\
 &\quad + T \cos\theta \cos^2\varphi (F_{fl} - F_{fr}) + T \cos\theta \cos^2\varphi (F_{rl} - F_{rr}) \\
 M_u \ddot{z}_{uf} &= K_{sf}z + K_{sf}L_f \sin\theta + K_{sf}T \sin\varphi - (K_{uf} + K_{sf})z_{uf} + K_{uf}z_{rl} - M_u g - F_{fl} \cos\theta \cos\varphi \\
 M_u \ddot{z}_{ur} &= K_{sf}z + K_{sf}L_f \sin\theta - K_{sf}T \sin\varphi - (K_{uf} + K_{sf})z_{ur} + K_{uf}z_{fr} - M_u g - F_{fr} \cos\theta \cos\varphi \\
 M_u \ddot{z}_{ur} &= K_{sr}z - K_{sr}L_r \sin\theta + K_{sr}T \sin\varphi - (K_{ur} + K_{sr})z_{ur} + K_{ur}z_{rl} - M_u g - F_{rl} \cos\theta \cos\varphi \\
 M_u \ddot{z}_{ur} &= K_{sr}z - K_{sr}L_r \sin\theta - K_{sr}T \sin\varphi - (K_{ur} + K_{sr})z_{ur} + K_{ur}z_{rr} - M_u g - F_{rr} \cos\theta \cos\varphi
 \end{aligned} \quad (1)$$

where,  $z, \theta, \varphi$  are the heave position, pitch angle and roll angle of the sprung mass;  $z_{uf}, z_{ur}, z_{ur}, z_{ur}$  are the unsprung mass heights of four corners;  $z_{rl}, z_{fr}, z_{rl}, z_{rr}$  are the terrain disturbance heights of four corners;  $M_s, M_u$  are the sprung mass and unsprung mass respectively;  $K_{sf}, K_{sr}, K_{uf}, K_{ur}$  are the stiffness of front springs, rear springs, front tires and rear tires, respectively;  $I_p, I_r$  are the pitch axis and roll axis moment of inertia;  $L_f, L_r$  are the distance from C.G. to the front axle and rear axle,  $T$  is the half track width of the sprung mass;  $g$  is the acceleration due to gravity;  $F_{fl}, F_{fr}, F_{rl}, F_{rr}$  are the active forces generated by the electromagnetic actuators.

Assume that the nominal values of mass and inertia of sprung mass subsystem are  $M_{s0}, I_{p0}, I_{r0}$  the range of the variation of the parameters of the active suspension system are bounded as follows

$$\begin{aligned}
 M_{s \min} &< M_s = M_{s0} + \Delta M_s < M_{s \max} \\
 I_{p \min} &< I_p = I_{p0} + \Delta I_p < I_{p \max} \\
 I_{r \min} &< I_r = I_{r0} + \Delta I_r < I_{r \max}
 \end{aligned}$$

where  $\Delta M_s, \Delta I_p, \Delta I_r$  are their deviations,  $M_s, I_p, I_r$  are the actual values.

The nominal model of the sprung mass subsystem is obtained without unmodeled dynamic and external disturbances as follow:

$$\begin{cases}
 \ddot{z} = -\frac{2(K_{sf} + K_{sr})}{M_{s0}}z - \frac{2(K_{sf}L_f - K_{sr}L_r)}{M_{s0}}\sin\theta + \frac{1}{M_{s0}}u_z \\
 \ddot{\theta} = -\frac{2(K_{sf}L_f - K_{sr}L_r)\cos\theta}{I_{p0}}z - \frac{2(K_{sf}L_f^2 + K_{sr}L_r^2)\cos\theta}{I_{p0}}\sin\theta + \frac{1}{I_{p0}}u_\theta \\
 \ddot{\varphi} = -\frac{2(K_{sf} + K_{sr})T^2 \cos\varphi}{I_{r0}}\sin\varphi + \frac{1}{I_{r0}}u_\varphi
 \end{cases} \quad (2)$$

$$\begin{aligned}
 \text{where } u_z &= (F_{fl} + F_{fr} + F_{rl} + F_{rr}) \cos\theta \cos\varphi, \\
 u_\theta &= [L_f(F_{fl} + F_{fr}) - L_r(F_{rl} + F_{rr})] \cos^2\theta \cos\varphi, \\
 u_\varphi &= T(F_{fl} - F_{fr} + F_{rl} - F_{rr}) \cos\theta \cos^2\varphi.
 \end{aligned}$$

### III. INTEGRAL SLIDING MODE CONTROL AND ITS APPLICATION ON ACTIVE SUSPENSION SYSTEM

#### 1. ISMC scheme

The design concept of ISMC is that a discontinuous term is added to the existing feedback controller for a nominal plant model to ensure the desired performance despite parametric uncertainty and external disturbances. The design procedure of ISMC [10] is described briefly as follow.

An actual system with uncertainty conditions is

$$\dot{x} = f(x) + B(x)u + h(x, t) \quad (3)$$

and its ideal closed-loop system is

$$\dot{x}_0 = f(x_0) + B(x_0)u_0 \quad (4)$$

where  $x \in R^n$  is the state vector and  $u \in R^m$  is the control input vector,  $B(x)$  is an appropriate matrix with  $\text{rank}B(x) = m$ ,  $h(x, t)$  represents uncertainty conditions such as parameter variations, unmodeled dynamics and external disturbances;  $x_0$  is the state trajectory of the ideal system under ideal feedback control  $u_0$ . Assume that the perturbation  $h(x, t)$  is bounded and is of the form:

$$|h_i(x, t)| \leq h_i^u(x, t), \quad i = 1, \dots, n$$

where  $h_i^u(x, t)$  is the upper bound of  $h_i(x, t)$  and being known positive scalar functions.

To ensure the sliding surfaces from the initial time instant, i.e.  $x(0) = x_0(0)$ , the control law is redesigned as follow

$$u = u_0 + u_d \quad (5)$$

with  $u_d \in R^m$  is designed to reject the perturbation term  $h(x, t)$ .

An integral item is added to the conventional sliding manifold to consist the novel sliding surface

$$s = s_0(x) + s_d$$

with  $s, s_0(x), s_d \in R^m$ .

The first control part  $u_0$  is to ensure the ideal system trajectory; the second control part  $u_d$  is to reject the perturbation and enforce the sliding manifolds that

$$\dot{s} = \dot{s}_d + \hat{s}_0(x) = \frac{\partial \hat{s}_0}{\partial x} [f(x) + B(x)u_0 + B(x)u_d + h(x, t)] + \dot{s}_d = 0$$

The integral item is defined to meet the requirement  $s(0) = 0$

$$\dot{s}_d = -\frac{\partial \hat{s}_0}{\partial x} [f(x) + B(x)u_0], \quad s_d(0) = -s_0(x(0))$$

The control law  $u_d$  is defined to enforce sliding mode along the manifold via discontinuous

$$u_d = -M(x) \text{sign}(s) \quad (6)$$

where  $M(x)$  is the control gain and selected as scalar function or even a constant to simplify the control design.

The Lyapunov function is defined as  $V = \frac{1}{2} s^T s$ , the time derivative of  $V$  is  $\dot{V} = s^T \dot{s}$ . The goal of the control is enforce the sliding mode to the desired performance, i.e.  $s = 0$ , this lead  $V$  converge to zero, hence its time derivative  $\dot{V}$  should be  $\dot{V} \leq 0$ , the sliding surface  $s$  and its time derivative  $\dot{s}$  have different sign. The time derivative of  $s$  can be calculated as

$$\dot{s} = \frac{\partial s_0}{\partial x} [B(x)u_d + h(x,t)] = \frac{\partial s_0}{\partial x} [-B(x)M(x)\text{sign}(s) + h(x,t)] < 0$$

The above inequality is guaranteed by  $M(x) > h''(x,t)$ ,

i.e.  $M_i(x) > h_i''(x,t)$ ,  $i=1, \dots, m$ .

## 2. Application of ISMC on active suspension system

The main goal of the active damper system is to isolate the vehicle body from road irregularities. Hence, for the state equations of the system, the control goal is to keep the height, pitch and roll angle of the car body constant.

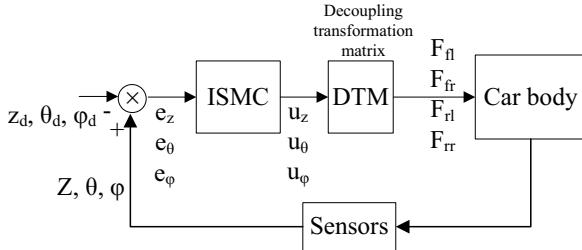


Fig.2: ISMC control scheme

Let  $z_d, \theta_d, \varphi_d$  be the desired value of  $z, \theta, \varphi$ , the error vector between the actual and desired values is defined as  $e^T = [z_e \ \theta_e \ \varphi_e] = [z - z_d \ \theta - \theta_d \ \varphi - \varphi_d]$ . The control law of the nominal sprung mass subsystem is demanded to converge the error vector  $e$  to zero at the well-designed trajectory, hence the closed-loop system obtains its desired dynamic performance. The ideal state-error trajectory of the nominal sprung mass subsystem is given

$$\begin{aligned} \ddot{z}_e + k_{zd}\dot{z}_e + k_{zp}z_e &= 0 \\ \ddot{\theta}_e + k_{\theta d}\dot{\theta}_e + k_{\theta p}\theta_e &= 0 \\ \ddot{\varphi}_e + k_{\varphi d}\dot{\varphi}_e + k_{\varphi p}\varphi_e &= 0 \end{aligned} \quad (7)$$

where  $k_{zd}, k_{zp}, k_{\theta d}, k_{\theta p}, k_{\varphi d}, k_{\varphi p}$  and  $k_{\varphi d}$  are the appropriate parameters to stable the plant and achieve the desired dynamic performance.

The first part of control vector  $u_0$  is obtained from (2) and (5) as

$$u_0 = \begin{bmatrix} u_{z0} \\ u_{\theta 0} \\ u_{\varphi 0} \end{bmatrix} = \begin{bmatrix} M_{s0} \left( \frac{2(K_{sf} + K_{sr})}{M_{s0}} z + \frac{2(K_{sf}L_f - K_{sr}L_r)}{M_{s0}} \sin \theta \right) + \ddot{z}_d - k_{zd}\dot{z}_e - k_{zp}z_e \\ I_{p0} \left( \frac{2(K_{sf}L_f - K_{sr}L_r) \cos \theta}{I_{p0}} z + \frac{2(K_{sf}L_f^2 + K_{sr}L_r^2) \cos \theta}{I_{p0}} \sin \theta \right) + \ddot{\theta}_d - k_{\theta d}\dot{\theta}_e - k_{\theta p}\theta_e \\ I_{r0} \left( \frac{2(K_{sf} + K_{sr})T^2 \cos \varphi}{I_{r0}} \sin \varphi \right) + \ddot{\varphi}_d - k_{\varphi d}\dot{\varphi}_e - k_{\varphi p}\varphi_e \end{bmatrix} \quad (8)$$

The integral type sliding manifold of the sprung mass subsystem is defined as

$$s = \begin{bmatrix} s_z \\ s_\theta \\ s_\varphi \end{bmatrix} = \begin{bmatrix} s_{z0} + s_{zd} \\ s_{\theta 0} + s_{\theta d} \\ s_{\varphi 0} + s_{\varphi d} \end{bmatrix} \quad (9)$$

in which the nominal sliding surface  $s_0$  is selected as

$$s_0 = \begin{bmatrix} s_{z0} \\ s_{\theta 0} \\ s_{\varphi 0} \end{bmatrix} = \begin{bmatrix} \dot{z}_e + \lambda_z z_e \\ \dot{\theta}_e + \lambda_\theta \theta_e \\ \dot{\varphi}_e + \lambda_\varphi \varphi_e \end{bmatrix}$$

and the integral item  $s_d$  is designed to enforce the sliding mode at the initial time instant

$$\dot{s}_d = \begin{bmatrix} \dot{s}_{zd} \\ \dot{s}_{\theta d} \\ \dot{s}_{\varphi d} \end{bmatrix} = - \begin{bmatrix} s_{z0} \\ s_{\theta 0} \\ s_{\varphi 0} \end{bmatrix} = \begin{bmatrix} -\ddot{z}_d + (k_{zd} - \lambda_z)\dot{z}_e + k_{zp}z_e \\ -\ddot{\theta}_d + (k_{\theta d} - \lambda_\theta)\dot{\theta}_e + k_{\theta p}\theta_e \\ -\ddot{\varphi}_d + (k_{\varphi d} - \lambda_\varphi)\dot{\varphi}_e + k_{\varphi p}\varphi_e \end{bmatrix}, \begin{bmatrix} s_{zd}(0) \\ s_{\theta d}(0) \\ s_{\varphi d}(0) \end{bmatrix} = - \begin{bmatrix} s_{z0}(0) \\ s_{\theta 0}(0) \\ s_{\varphi 0}(0) \end{bmatrix}$$

The time derivative of sliding surface  $s$  is

$$\dot{s} = \begin{bmatrix} \dot{s}_z \\ \dot{s}_\theta \\ \dot{s}_\varphi \end{bmatrix} = \begin{bmatrix} \left( \frac{M_{s0}}{M_s} - 1 \right) (\ddot{z}_d - k_{zd}\dot{z}_e - k_{zp}z_e) + \frac{1}{M_s} u_{zd} + \frac{1}{M_s} d_z \\ \left( \frac{I_{p0}}{I_p} - 1 \right) (\ddot{\theta}_d - k_{\theta d}\dot{\theta}_e - k_{\theta p}\theta_e) + \frac{1}{I_p} u_{\theta d} + \frac{1}{I_p} d_\theta \\ \left( \frac{I_{r0}}{I_r} - 1 \right) (\ddot{\varphi}_d - k_{\varphi d}\dot{\varphi}_e - k_{\varphi p}\varphi_e) + \frac{1}{I_r} u_{\varphi d} + \frac{1}{I_r} d_\varphi \end{bmatrix} \quad (10)$$

To ensure that the well-defined Lyapunov function  $V = \frac{1}{2} s^T s$  has negative time derivative  $\dot{V} = s^T \dot{s} \leq 0$ , the discontinuous control law is decided as

$$u_d = \begin{bmatrix} u_{zd} \\ u_{\theta d} \\ u_{\varphi d} \end{bmatrix} = \begin{bmatrix} M_{s0} (-K_z z_e \text{sign}(z_e s_z) - M_z \text{sign}(s_z)) \\ I_{p0} (-K_\theta \theta_e \text{sign}(\theta_e s_\theta) - M_\theta \text{sign}(s_\theta)) \\ I_{r0} (-K_\varphi \varphi_e \text{sign}(\varphi_e s_\varphi) - M_\varphi \text{sign}(s_\varphi)) \end{bmatrix} \quad (11)$$

where

$$\begin{aligned} K_z &> \text{Max} \left\{ \left| 1 - \frac{M_s}{M_{s0}} \right| k_{zp}, M_z > \text{Max} \left\{ \left| 1 - \frac{M_s}{M_{s0}} \right| (\ddot{z}_d - k_{zd}\dot{z}_e) + \frac{1}{M_{s0}} d_z \right\}, \right. \\ K_\theta &> \text{Max} \left\{ \left| 1 - \frac{I_p}{I_{p0}} \right| k_{\theta p}, M_\theta > \text{Max} \left\{ \left| 1 - \frac{I_p}{I_{p0}} \right| (\ddot{\theta}_d - k_{\theta d}\dot{\theta}_e) + \frac{1}{I_{p0}} d_\theta \right\}, \right. \\ K_\varphi &> \text{Max} \left\{ \left| 1 - \frac{I_r}{I_{r0}} \right| k_{\varphi p}, M_\varphi > \text{Max} \left\{ \left| 1 - \frac{I_r}{I_{r0}} \right| (\ddot{\varphi}_d - k_{\varphi d}\dot{\varphi}_e) + \frac{1}{I_{r0}} d_\varphi \right\}. \end{aligned}$$

The chattering phenomenon may be alleviated by two methods: (i) low-pass filter and (ii) saturation function. A LPF module with the form of  $\frac{1}{0.02s + 1}$  is used to smooth the discontinuous control generated by (11); the saturation function introduced here to replace the sign function and or even avoided

$$\text{sat}(s) = \begin{cases} 1, s > \Delta \\ s / \Delta, -\Delta < s < \Delta \\ -1, s < -\Delta \end{cases}$$

where  $\Delta$  is the boundary layer which is a positive constant.

In addition, the transformation of inputs  $u_z, u_\theta, u_\varphi$  and the actuators output forces is represented here:

$$\begin{pmatrix} u_z \\ u_\theta \\ u_\varphi \end{pmatrix} = \cos \theta \cos \varphi \begin{pmatrix} 1 & 1 & 1 & 1 \\ L_f \cos \theta & L_r \cos \theta & -L_r \cos \theta & -L_r \cos \theta \\ T \cos \varphi & -T \cos \varphi & T \cos \varphi & -T \cos \varphi \end{pmatrix} \begin{pmatrix} F_\beta \\ F_{fr} \\ F_{rl} \\ F_{rr} \end{pmatrix}$$

The required generated forces of electromagnetic actuators are

$$\begin{pmatrix} F_{\beta} \\ F_{fr} \\ F_{rl} \\ F_{rr} \end{pmatrix} = \frac{1}{\cos\theta\cos\varphi} \begin{pmatrix} \frac{L_r}{2(L_f+L_r)} & \frac{1}{2(L_f+L_r)\cos\theta} & \frac{1}{4T\cos\varphi} \\ \frac{L_r}{2(L_f+L_r)} & \frac{1}{2(L_f+L_r)\cos\theta} & \frac{-1}{4T\cos\varphi} \\ \frac{L_f}{2(L_f+L_r)} & \frac{-1}{2(L_f+L_r)\cos\theta} & \frac{1}{4T\cos\varphi} \\ \frac{L_f}{2(L_f+L_r)} & \frac{-1}{2(L_f+L_r)\cos\theta} & \frac{-1}{4T\cos\varphi} \end{pmatrix} \begin{pmatrix} u_z \\ u_\theta \\ u_\varphi \end{pmatrix}$$

#### IV. SIMULATION RESULTS

The parameters of the active suspension system of the vehicle are shown in Table 1. The parameters of the SMC controller are listed in Table 2. Without generality, most of the gains are selected to be identical for simplicity.

**Table 1: System parameters**

Parameter	Value
$M_s$	1500 (kg)
$M_u$	59 (kg)
$K_{sf}$	20000 (N/m)
$K_{sr}$	21000 (N/m)
$K_u$	190000 (N/m)
$I_p$	460 ( $\text{kg}\cdot\text{m}^2$ )
$I_r$	2160 ( $\text{kg}\cdot\text{m}^2$ )
$L_f$	1.4 (m)
$L_r$	1.7 (m)
$T$	0.8 (m)

**Table 2: Controller parameters**

Parameter	Value
$k_{zp}, k_{\theta p}, k_{\varphi p}$	1
$k_{zd}, k_{\theta d}, k_{\varphi d}$	10
$K_z, K_\theta, K_\varphi$	1
$M_z, M_\theta, M_\varphi$	4.5, 3.5, 6
$\lambda_z, \lambda_\theta, \lambda_\varphi$	5
$\Delta_z, \Delta_\theta, \Delta_\varphi$	0.001

The road irregularities contain an isolated trapezoidal bump and sinusoidal two-track roads, as illustrated in Fig. 3(a) and 4(a). The time delay  $\tau$  could be calculated under certain velocity, due to the track length between the front and rear axles which is  $L = L_f + L_r = 3.1\text{m}$ . Once the longitudinal velocity of the vehicle is assumed  $v = 20\text{m/s}$ , the time delay can be calculated as  $\tau = L/v = 0.155\text{s}$ , which is shown in Fig.3 (a). The response of car body indicates that the ISMC can absorb the road irregularity rapidly, compared with PID control as shown in Fig.3 (b).

The wavelength of the sinusoidal road is assumed to be 20m and the car runs over it under constant speed  $v = 20\text{m/s}$ . Fig. 4 (b) shows the response of the car body under sinusoidal road which the phase-shift angle equals

$90^\circ$ . The results show that ISMC greatly reduces the oscillations due to the continuous sinusoidal road without the knowledge of the car body and road disturbance. It is convinced that the active suspension system with SMC controller improves the ride comfort. The corresponding generated forces are shown in Fig. 3(c) and 4(c). The required maximum forces are about 2000N, it is available for electrical actuator to generate the active force.

The actual parameters of the active suspension system, such as mass and inertia of sprung mass subsystem, will vary with different load. The robustness of the ISMC controller can guarantee the desired performances of the active suspension system with the variations of the system parameters reach up to 50%, i.e.  $\Delta M_s = \pm 0.5M_{s0}$ ,  $\Delta I_p = \pm 0.5I_{p0}$ ,  $\Delta I_r = \pm 0.5I_{r0}$ , as shown in Fig. 5.

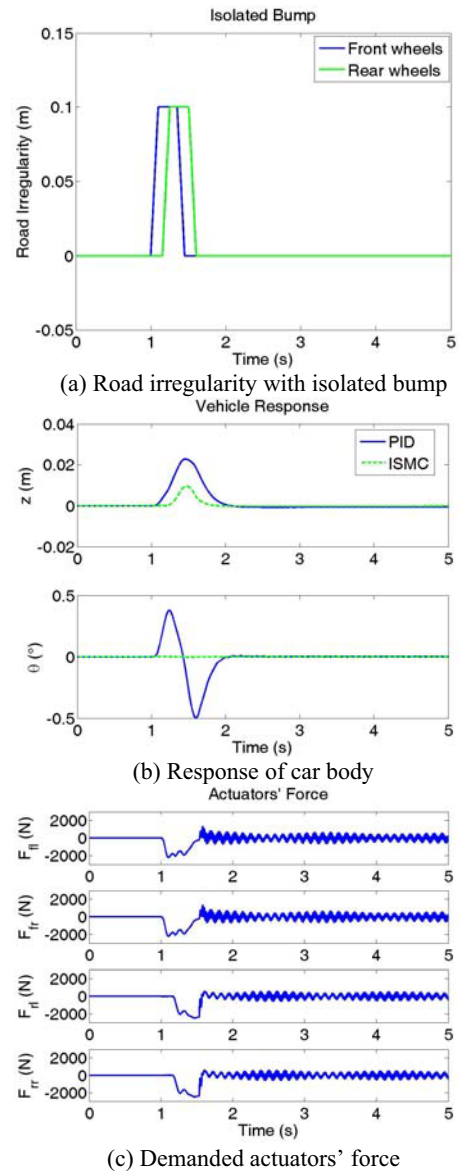
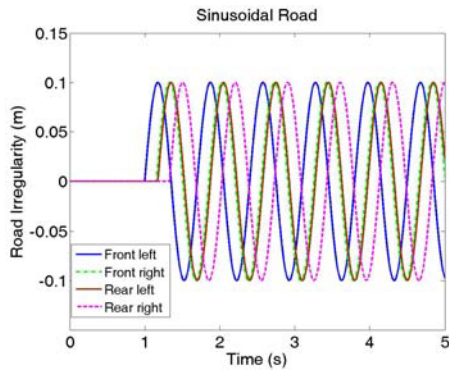
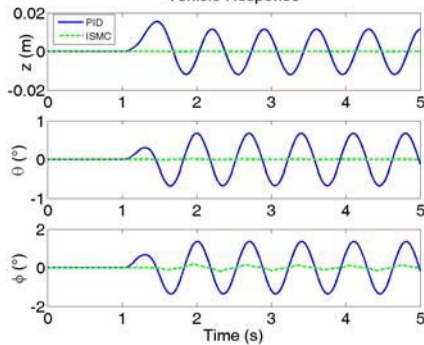


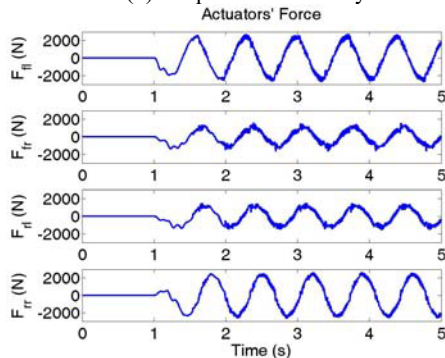
Fig.3: ISMC controller under isolated bump road



(a) Road irregularity with sinusoidal road



(b) Response of car body



(c) Demanded actuators' force

Fig.4: ISMC controller under sinusoidal road

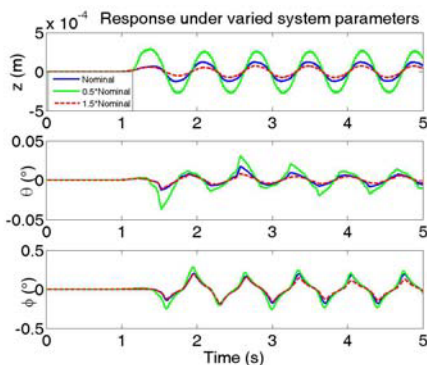


Fig.5: Response under sinusoidal road with varied system parameters

## V. CONCLUSION

An accurate nonlinear active suspension model is developed in this paper, to track the accurate dynamic of the vertical motion of the vehicle. The decoupling transformation matrix is used to decouple the sprung subsystem and reduce the order of the whole system. Integral sliding mode control method is studied then to reject the road irregularities without the knowledge of the

vehicle. As shown in Section IV, the dynamic responses of car body with ISMC scheme can be enhanced greatly comparing with conventional PID controller, hence the ride comfort and safety can be improved despite road irregularities.

The required force generated is constrained within 2000N by optimizing the SMC controller, as shown in the simulation results. It is below the output range of present linear electrical actuators; hence developing of linear electrical actuators and equipping into the active suspension system for implementation are available in future.

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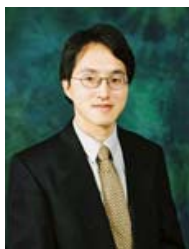
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## BIOGRAPHY



**J.K. Lin** received his BEng degree in 2003 from Huazhong University of Science and Technology and Master degree in 2007 from South China University of Technology, respectively. After receiving his Master degrees, he joined HK Polytechnic University as a Research Assistant. He is now pursuing PhD at the same university. His research interests are actuator drive, motion control and

applications of power electronics to electric vehicle.



**K.W.E. Cheng** obtained his BSc and PhD degrees both from the University of Bath in 1987 and 1990 respectively. Before he joined the Hong Kong Polytechnic University in 1997, he was with Lucas Aerospace, United Kingdom as a Principal Engineer.

He received the IEE Sebastian Z De Ferranti Premium Award (1995), outstanding consultancy award (2000), Faculty Merit award for best teaching (2003) from the University and

Silver award of the 16<sup>th</sup> National Exhibition of Inventions. He has published over 200 papers and 7 books. He is now the professor and director of Power Electronics Research Centre.



**Zhu Zhang** obtained his MSc degree from South China University of Technology, Guangzhou, China, in 2004. He is currently working toward the Ph.D. degree at the Department of Electrical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong. His research interests include motor design and power electronics.



**Dr. N.C. Cheung** received the B.Sc. degree from the University of London, London, U.K., in 1981, the M.Sc. degree from the University of Hong Kong, Kowloon, Hong Kong, in 1987, and the Ph.D. degree from the University of New South Wales, Kensington, NSW, Australia, in 1996.

He is currently working in the Department of Electrical Engineering, the Hong Kong Polytechnic University, Kowloon, Hong Kong. His research interests are motion control, actuators design, and power electronic drives.



**X. D. Xue** received the B.Eng. degree from Hefei University of Technology, Hefei, China, in 1984, the M.Eng. degree from Tianjin University, Tianjin, China, in 1987, and the Ph.D. degree from the Hong Kong Polytechnic University, Kowloon, Hong Kong, in 2004, all in electrical engineering.

He was a Lecturer and an Associate Professor with the Department of Electrical Engineering, Tianjin University, from 1987 to 2001, where he was engaged in teaching and research. He is

currently a Research Fellow with the Department of Electrical Engineering, Hong Kong Polytechnic University. He is the author of over 70 published papers. His research interests include electrical machines, electrical drives, and power electronics. His current research is focused on electric machines and drives applied to electric vehicles and wind-power generations.



**M.K. Wong** received his BEng degree in 2007 and MSc degree in 2010 from The Hong Kong Polytechnic University. He has joined the Power Electronic Research Centre of the same University as a Research Assistant since 2007. His research interests are power converters, high power battery chargers for electrical vehicles and battery management systems.



**Y. J. Bao** obtained his Bachelor's degrees in automatic department from Northeastern University, Qinhuangdao, China, in 2000. He received his Master's degree in automatic department from Shanghai University, Shanghai, China, in 2003. Currently, he is an Research Associate in the department of Electrical Engineering at the Hong Kong Polytechnic University, Hong Kong, China.

His research interests include motor & generator control and drive.