Survey of Modeling Methods for Flux Linkage of Switched Reluctance Motor

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Abstract–Switched reluctance motors (SRMs) have attracted many researchers' and engineers' attention, due to their advantages. However, its electromagnetic behaviors are very complicated and hard to be modeled. Therefore, modeling of flux linkage plays an important role for this type of machine. In this paper, some existed modeling methods of flux linkage for SRMs are surveyed. These modeling methods are fallen into seven classes according to their different features. Also, the advantages and limitations of these methods are discussed. Some considerations for selecting a proper modeling method are introduced form simulations and controller design, etc. Finally, the modeling methods are compared and evaluated based on the considerations.

Keywords–Switched reluctance motor, flux linkage modeling, nonlinear, modeling method.

I. INTRODUCTION

Recently Switched Reluctance Motors (SRMs) have been obtained more and more attention for adjustable speed applications due to their simple structures, low cost and good reliability in harsh environments. However, the simple salient structure of SRM results in highly nonlinear magnetic behaviors, which make it difficult to determine the performances of SRM. Therefore, modeling of flux linkage of SRMs plays an important role on controller design and performance analysis, etc.

Over the years, several flux linkage modeling techniques for SRMs have been proposed. Most direct approach is lookup table, which stores the entire flux linkage characteristics as the model [1-2]. Another simple modeling scheme is the piecewise method that can be subdivided into linear method [3-5] and nonlinear method [6-8] according to their approximation functions. Following those, some techniques of two dimensional curve fitting were proposed for the flux linkage approximation over the whole region. Under this curve fitting category, three methods are commonly used; they are the trigonometric approximation method [9-10], the polynomial approximation method [11-13] and the exponential description function method [14-16]. With the increasing development in AI techniques, some AI based approximation methods have also been applied in the flux linkage modeling [17-20]. In this paper, a survey about the existed modeling methods for the various flux linkage of SRM is investigated.

The organization of this paper is as follows. The nonlinear characteristics between the flux linkage and position and phase current are described in Section II. In Section III, various existed modeling methods for the flux linkage of SRMs are classified and discussed. In Section IV, some considerations during the modeling are introduced for simulation and controller design, etc. After that, the modeling methods are compared and evaluated in Section V. Concluding remarks are given in Section VI.

II. Characteristics of Flux Linkage of SRM

The fundamental electromagnetic behaviors of SRMs can be described by the voltage balance equation (1). It can be reformatted as equation (2) because the phase flux linkage is a function of its phase inductance and phase current. Furthermore, a phase flux linkage can be calculated by using equation (3) from the viewpoint of measurement. And the produced torque for one phase can be described by equation (4).

$$v = ri + \frac{d\lambda}{dt} \tag{1}$$

$$v = ri + L\frac{di}{dt} + i\frac{\partial L}{\partial i}\frac{di}{dt} + i\frac{\partial L}{\partial \theta}\omega \tag{2}$$

$$\lambda = \int_0^T (v - ri)dt \tag{3}$$

$$T = \frac{\partial}{\partial \theta}\int_0^I \lambda di \tag{4}$$

where v , i , r , λ and L denote terminal voltage, winding current, winding resistance, winding flux linkage and phase inductance; θ, ω and T denote rotor position, rotor speed and electromagnetic torque, respectively. The second term on the right hand of (2) is referred as back EMF.

It can be seen that the electromagnetic behavior of SRMs can not be represented by some simple equations as those for other types of machines. All of the equations include the phase flux linkage. Therefore, an easy used model of the flux linkage is the key to describe and predict the behaviors of SRMs so that perform simulations and design controllers for this machine.

Fig. 1 shows a general 2D characteristic between the phase flux linkage and phase current and rotor position. And its 3D mesh is shown in Fig. 2. It can be seen that the phase flux linkage at the unaligned position is linear to phase current while he flux linkage become saturated to phase current with position tending to aligned position. At the aligned position, the phase flux linkage is obviously saturated to phase current.

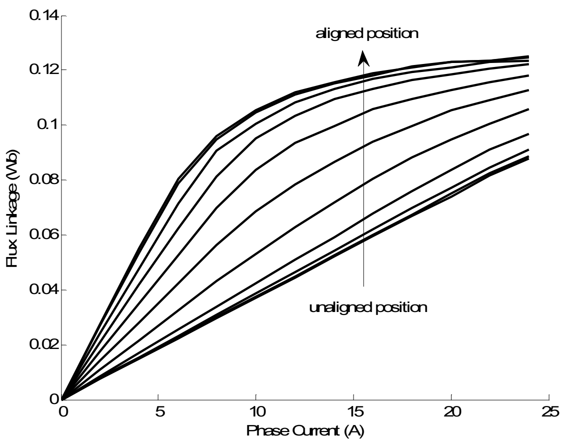

Fig. 1: A general 2D characteristic between phase flux linkage, phase current and position.

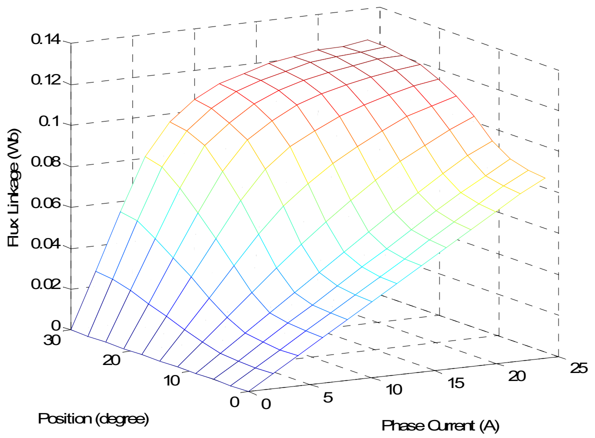

Fig. 2: A general 3D characteristic between phase flux linkage, phase current and position.

From the above two figures, it could be found that the characteristics of flux linkage are very complicated and hard to be precisely described by some simple methods.

III. Survey of Modeling Methods for Flux Linkage

In this section, existed various modeling methods for flux linkage of SRM would be surveyed in the following seven classes.

A. Lookup table

Lookup table (LUT) is a most common method used to describe nonlinear relationships in that it can be implemented in some low power processors. This method directly makes use of experimental data of flux linkage vs. phase current vs. rotor position to find some characteristic points hence represent their nonlinear relationships [1-2].

The precision of this method often relies on the density of characteristic points. However, building up a lookup table, especially for a high precision lookup table, would require considerable time to process a large amount of experimental data. Furthermore, high volume of lookup table requires large memory storage.

B. Piecewise linear approximation

Piecewise linear approximation (PLA) is one of the simplest flux linkage approximations for SRMs [3-5]. Instead of modeling the whole flux linkage characteristic directly, it divides the flux linkage relationship into different sections bases on its characteristic plane. Each section is represented by a linear model. Generally, the whole flux linkage characteristic includes three sections as linear, saturated and highly saturated regions.

Modeling with linear functions introduces extra modeling error. However, this approximation method is simple and inexpensive for practice implementation.

C. Piecewise nonlinear approximation

Similar to piecewise linear approximation, this method also builds the flux linkage model of SRM by separating the magnetic characteristics into three different sections but it adopts nonlinear approximation. Two sections represent the boundary of the magnetic characteristics corresponding to the magnetization curve for the unaligned and aligned position. The other describes the rest region corresponding to the magnetization curves in midway. This method has been investigated in several research works [6-8].

Piecewise nonlinear approximation (PNA) is still a relatively simple modeling solution for SRMs. The accuracy of this method mainly relies on the approximation of the magnetization curves in the middle section.

D. Trigonometric approximation

For a trigonometric approximation (TA), flux linkage or phase inductance is expressed with rotor position and phase current by using Fourier series [9-10]. The rotor position is treated as the variable of this Fourier series. The coefficients of the series are the functions of phase current. They can be found in a least square sense. Different from the piecewise approximation scheme, this method adopts a two dimensional curve family to approximate the whole region of flux linkage characteristics.

This method can accurately approximate the flux linkage characteristics. While the main disadvantage for this method is the presence of trigonometric functions because the computation of sine and cosine terms is complex and time-consuming.

E. Polynomial approximation

Similar to Fourier series, polynomials have the ability to fit nonlinear functions. Hence, polynomial approximation (PA) is often used to model the magnetic characteristics of

SRMs [11-13]. For this method, the flux linkage is firstly divided into many separate regions then each region is represented by a polynomial of position and current.

This method can offer high modeling accuracy and is more preferred than the previous one because no trigonometric calculation is required. To ensure high accuracy, the flux linkage data need to be divided into several regions. However, too many regions will enlarge the storage for its coefficients and slow down its calculation speed.

F. Exponential description function

Relying on the fact that the flux linkage vs. current plot of a SRM is very similar to an exponential function, the exponential description function (EDF) is a model tailored for SRMs. The flux linkage is described as an exponential function of current and position depended function. This method was firstly proposed in [11] and was improved to represent the flux linkage characteristics in [14-16].

This method does not split the magnetization curve into different regions and avoids the continuity problem. Similar to the flaws of trigonometric approximation, it also suffers from the inefficiently calculating problem of trigonometric and exponential functions. Ripples will appear when the higher order terms are neglected.

G. Artificial intelligence based method

By utilizing the abilities of approximating nonlinear functions, Artificial intelligence (AI) based method has attracted more and more researchers' attention for the flux linkage modeling. Different from the analytical approaches mentioned above, this method requires no mathematical model. Under this category, two techniques are mainly adopted, referred as hidden-layer based Artificial Neural Networks (ANN) [17-18] and rule-set based Fuzzy Logic (FL) [19-20].

These methods can provide very high precision for the flux linkage approximation of SRMs. However, the training of ANN or FL model requires a large amount of given data. Due to the complex computation process, AI methods require high performance microprocessor or Digital Signal Processor (DSP).

IV. Considerations on Flux Linkage Modeling of SRM

There are a number of existed modeling methods. These methods have their advantages and limitations. Therefore, the special situations or purposes for modeling should be considered, such as simulations, controller design, etc. Generally speaking, the modeling of flux linkage is mainly for the purpose of simulations, real-time control. Therefore, some requirements should be considered as follows,

i) Minimum data size. Ideally, the flux linkage characteristics should be described by just a few parameters. In this way the model is more manageable by the processor.

ii) Efficiency of computation. The applied model should be suitable for high-speed implementation. Model which requires less processing and puts less demand on the computation time of the processor is much preferred.

iii) Reasonable accuracy. Accuracy should be adequate to describe the magnetic characteristics during calculating control signals.

V. Comparisons on the Modeling Methods of SRMs

According to the considerations of flux linkage modeling, the existed modeling methods for SRMs can be evaluated as in the following table.

Table 1: comparisons on the modeling methods

Method	Data Size	Efficiency	Complexity	Accuracy
LUT	Big	High	Low	Low
PLA	Medium	High	Low	Low
PNA	Medium	Medium	Medium	Medium
TA	Low	Medium	Medium	Medium
PA	Low	Medium	Medium	Medium
EDF	Low	Medium	Medium	Medium
AI	Big	Low	High	High

The LUT and PLA are the simplest in all of the methods but the accuracy can be obtained is relative low. The AI method can obtain a model with very high accuracy but its realization is relative complex. And the other modeling methods in this paper are with medium accuracy and medium complexity. It can be seen that these methods have their advantages and limitations. In applications, a proper modeling method needs to be selected according to the requirements.

VI. Conclusion

In this paper, some existed modeling methods for flux linkage of SRMs are surveyed. These modeling methods are cataloged into seven classes according to their features. Also, the advantages and limitations of these methods are investigated. In addition, some considerations for selecting a proper modeling method are introduced for simulation and controller design, etc. Finally, the modeling methods are compared and evaluated.

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