# Real-time On-line Parameter Estimation of Linear Switched Reluctance Motor

N.C. Cheung, J.F. Pan, Jin-quan Li

*Abstract*—The on-line least-square identification method is applied for the linear switched reluctance motor (LSRM) based closed-loop control system. Since switched reluctance motors have severe nonlinear characteristics, on-line system identification for model parameters is the basis for precise modeling and effective control algorithms. By on-line parameter identification of LSRM with least-square scheme, the parameter characteristics can be predicted despite of operation variations. The experimental results demonstrate the identification scheme has fast response and converging speed and the capability of effective parameter prediction under different conditions.

*Index terms*— dSPACE 1104, least-square, LSRM, on-line identification, parameter convergence, PD controller, switched reluctance.

## I. INTRODUCTION

Linear actuators are being increasingly considered for machine tool drives because they reduce the need for mechanical subsystems of gears and rotary-to-linear motion converters, such as lead screw. The LSRM is vividly obtained from its rotary counterpart by cutting along the shaft over its radius, both the stator and rotor and then rolling them out and it has a mechanically robust structure [1]. The LSRM is difficult to control and its output has a higher force ripple because of the complexity and nonlinearity of the LSRM magnetic circuit, which is difficult to precisely model and simulate.

The parameters of LSRM change with environment. So it is essential to identify the parameters of LSRM for implementation of control algorithms. The least-square method is a basic technique for parameter estimation which has a simple on-line structure [2]. Therefore the least-square method is ideal for parameter identification of LSRM model in this paper.

#### II. THE CONFIGURATION OF LSRM

The construction of LSRM is shown in Fig.1. The three-phase LSRM has a configuration with an active (with windings) translator and a passive (without windings) stator, and both are laminated with 0.5 silicon-steel plates. The machine is simple to manufacture, mechanically robust, and has lower eddy current losses as the flux is in the same direction as lamination. A 1  $\mu$ m resolution linear optical encoder is integrated in the LSRM system to observe the motion profile of the moving translator and provide the feedback position information. The mechanical parameters of the LSRM are listed in Table I.

TABLE I

MECHANICAL PARAMETER OF THE LSRM

Mass of translator	3 Kg
Mass of stator	17 Kg
Motor length	120 mm
Pole width	6 mm
Slot width(z)	6 mm
Encoder resolution	1 µm
Air gap width(z)	0.4 mm

Fig. 1. The instruction of LSRM

## III. MODELING OF LSRM

The LSRM system has a highly nonlinear characteristic because of its nonlinear flux behavior. An elementary equivalent circuit for the LSRM can be derived neglecting the mutual inductance between the phases, and the fundamental equations of LSRM are the voltage balancing formulas. The applied voltage to a phase is equal to the sum

The authors would like to thank the Hong Kong Research Grants Council for the sponsoring of this research project under the project code PolyU 5140/07E and PolyU project under account code A-PH45; the authors would also like to thank Guangdong Natural Science Fund 2008225 and Shenzhen government fund 08cxy-29 for their support.

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of the resistive drop and the rate of the flux-linkages given as [3]:

$$V = R_s i + \frac{d\lambda(x,i)}{dt} \tag{1}$$

where  $R_s$  is the resistance per phase, x is displacement, and  $\lambda$  is the flux linkage per phase given by,

$$\lambda = L(x, i)i \tag{2}$$

where L is the inductance dependent on the translator position and phase current. The phase voltage equation, then, is

$$V = R_s i + L(x, i) \frac{di}{dt} + i \frac{dx}{dt} \cdot \frac{dL(x, i)}{dx}$$
(3)

The force command of translator is given as:

$$f(i_{a}(t), i_{b}(t), i_{c}(t), x(t)) = \sum_{k=a}^{c} \frac{\partial \int_{0}^{i_{k}(t)} \lambda_{k} \cdot d\tau_{k}(t)}{\partial x(t)}$$

$$= M \frac{d^{2}x(t)}{dt^{2}} + Bv \frac{dx(t)}{dt} + f_{l}(t)$$
(4)

where f is the generated electromagnetic force,  $f_1(t)$  is the

external load force, M and B are the mass and friction constant, respectively.

## IV. PRINCIPLE OF THE LEAST SQUARE WITH EXPONENTIAL FORGETTING

It is assumed that the identification process is described by the signal-input, signal-output (SISO) system as,

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t)$$
(5)

where

$$\begin{cases} A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_n q^{-n} \\ B(q^{-1}) = b_0 + b_1 q^{-1} + \dots + b_n q^{-n} \end{cases}$$
(6)

The model is linear in the parameters and can be written as [3],

$$y(t) = \varphi^{T}(t-1)\theta + e(t)$$
(7)

where

$$\begin{cases} \theta^{T} = (a_{1} a_{2} \cdots a_{n} b_{0} b_{1} \cdots b_{n}) \\ \varphi^{T}(t) = (-y(t-1) \cdots - y(t-n)u(t) \cdots u(t-n)) \end{cases}$$
(8)

and  $t = n + 1, \dots, n + N$ , then the matrix  $\Phi(N)$  can be written as,

$$\Phi(N) = \begin{bmatrix} \varphi^T(n+1) \\ \vdots \\ \varphi^T(N) \end{bmatrix}$$
(9)

If the matrix  $\Phi^T \Phi$  is nonsingular, the least-square estimates of the parameters can be simply obtained,

$$\hat{\theta}_{LS} = (\Phi^T \Phi)^{-1} \Phi^T Y \tag{10}$$

The least-squares estimator with exponential forgetting is given by

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$$
(11)

$$\varepsilon(t) = y(t) - \varphi^{T}(t-1)\hat{\theta}(t-1)$$
(12)

$$K(t) = P(t-1)\varphi(t-1)(\lambda + \varphi^{T}(t-1)P(t-1)\varphi(t-1))^{-1}$$
(13)

$$P(t) = \left(I - K(t)\varphi^{T}(t-1)\right)P(t-1)/\lambda$$
(14)

where *P* is the covariance matrix and  $\lambda$  is forgetting factor. *K*(*t*) can be interpreted as the adjusting gain. If *K*(*t*)=0, the estimated parameter  $\theta$  converges to certain constants. The forgetting factor, which can be selected from 0 to 1, reflects the parameter converging rate. A smaller forgetting factor can result in faster converge speed of parameters with bigger ripples. The initial covariance matrix *P*(0) is selected as *rI*, which is a 2*n*-th unit matrix scaled by a positive scalar *r*, *r* is given as a big value as usual.

#### V. DESIGN OF LSRM IDENTIFICATION

A PD controller is applied to the linear switched reluctance motor for position control with real-time close-loop parameter estimation as the flow chart of real-time parameter estimations shown in Fig.2. According to the equation(4), the LSRM can be considered as a second-order system with the current tracking controller.  $q^{-1}$  is defined as the backward shift operator, the

second-order system can be written as the single-input single-output discrete model given by

$$A(q^{-1})y(t) = B(q^{-1})[u(t) + w(t)]$$
(13)

where

$$\begin{cases} A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} \\ B(q^{-1}) = b_0 q^{-1} + b_1 q^{-2} \end{cases}$$
(14)

The goal is to identify the value of  $a_1, a_2, b_0$  and  $b_1$ .



Fig. 2. The flow chart of real-time parameter estimations

## VI. EXPERIMENTAL IMPLEMENTATION

The experimental setup is shown in Fig.3. The host PC is used to target code into a dSPACE DS1104 DSP motion controller card [4]. The control algorithm is developed under the MATLAB/SIMULINK environment. For the current tracking amplifier [5], the current driver consists of three DA output with 90VDC voltage supplier. In the experiment, the sampling time is 0.001 s.



Fig. 3. The experimental setup

A. Square-wave Reference Input

Fig.4 shows the position tracking waveforms and its

corresponding control signal waveforms in the control of the PID controller. The input signal is square-wave with amplitude of 20 mm and frequency of 1 Hz. It can be seen that its performance is not identical between positive and negative half-cycle because of asymmetry of mechanical structures.



Fig. 4. Trajectory response of square-wave command

Fig.5 shows the parameters estimating of the LSRM system. Parameters  $a_1$ ,  $a_2$ ,  $b_0$  and  $b_1$  can converge to their stable values within 8 s.





Fig. 5. Parameter estimation of (a)a1,(b)a2,(c)b0,(d)b1

#### B. Sin-wave Reference Input

Fig.6 shows the tracking performance under the input signal of sin-wave with amplitude of 20 mm and frequency of 1 Hz.



Fig. 6. Trajectory response of sin-wave command

Fig.7 shows parameter estimation under sine-wave position command. It can be seen that the parameters converge to their stable value within 6s.





Fig. 7. Parameter estimation of (a)a1,(b)a2,(c)b0,(d)b1

## VII. CONCLUSION

This paper proposes an identification formula based the least-square method. The square-wave and sin-wave are applied to illustrate the convergence of estimation. Implementation result shows that the parameter can converge to stable value quickly, and has a good performance in tracking variety of environment. This result also proves the applicability of the algorithm for further advanced Algorithms.

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