Low cost displacement sensor at sub-micron precision based on 3×3 optical coupler for vibrating surface

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Abstract—A high precision displacement sensor is proposed in this paper. This sensor based on 3×3 optical coupler and analog circuits is linear incremental encoder type. Outputs of optical coupler are processed by analog circuits and generate displacement information. Low cost and high precision are main advantages of this sensor and analog circuits could enhance the maximum measurable velocity compared with digital signal processor (DSP) [1]. The resolution of this proposed sensor depends on wavelength of laser λ and the resolution is $\lambda/16 =$ 95.5nm. The error deviation is 1µm, which is measured during experiment.

Keywords-component: interferometer, displacement sensor, sub-micron precision, interference

I. INTRODUCTION

Feedback linear displacement sensor is one of importance components in linear motion feedback control. The accurate linear motion control usually requires a high precision feedback displacement sensor. Many engineers and scientists have designed many sensors for this purpose. One of the popular high precision sensors is optical interferometer. Reference [1-4] show some designs and applications of high precision interferometric displacement sensor. The resolution of interferometer depends on wavelength of light sources which is typically in micrometer.

One of the innovation ideas is using 3×3 optical coupler to generate displacement information. Reference [5] shows a displacement sensor using this idea and demonstrates displacement measurement of moving object with 50pm resolution successfully. However, the measurable range is only $15\mu m$. Reference [1] proposes a sensor using similar idea together with another type of light source which is high coherence light source. Although method shown in [1] has greatly improved in measurable range, the measurable velocity



Fig.1 Interferometric displacement sensor constructed with 3×3 coupler (solid triangles represent original light beams direction and arrows represent reflected light beams direction)

is too slow for practical application.

This paper is proposing a modified sensor using the 3×3 optical coupler together with real-time analog circuits for signal processing. The maximum velocity of vibrating target could be 77mm/s and resolution is 95.5nm.

II. THEORY

A. Optical circuit

Fig.1 shows the optical circuit of displacement based on 3×3 optical interferometric method [1]. The laser diode is connected to input fiber P₁ of 3×3 optical coupler through the optical isolator. Fibers P₂ and P₃ on another side of the coupler are then connected to a light reflector and collimator. Light reflector is reflecting the laser beam back to the coupler. The collimator allows the laser beam launch in air. The laser beam projecting on vibrating object is then reflected back and collected by collimator. Reflected light beam is then launching back to coupler. Reflected light beams inside P₂ and P₃ are then superimposed inside 3×3 coupler.

To explain the optical signal in an easier way, Fig.2 which is unfolded form of the interferometer with 3×3 coupler is given [1,6,7]. In the other word, Fig.2 is equivalent optical circuit of interferometer using 3×3 coupler.

Considering Fig.2 and assuming the laser diode generates the light beam with optical power A^2 , the light beam is launched into 3×3 coupler A. This light beam is then split into three equal intensity light beams and then launched into P₁, P₂, and P₃. The 3×3 coupler is governed by a set of equation as shown in (1). [1, 6, and 7]

Suppose the coupling coefficient between the jth and ith waveguide is $K_{ij} = K_{ji}$ (i,j = 1,2,3 and i \neq j). The complex amplitudes a_j of three waves in the coupler are governed by a set of linear differential equations,



Fig.2 Unfolded form of 3×3 interferometer



Fig.3 Simulation result optical signal in of equation 5 and 7. Given conditions $KL = 30^\circ$, $L_3 + L_c = L_2$ and λ is 1550nm. X-axis is position *D* of the vibrating target. [1]

$$\frac{da_{1}}{dz} + iK_{12}a_{2} + iK_{13}a_{3} = 0$$

$$\frac{da_{2}}{dz} + iK_{23}a_{3} + iK_{21}a_{1} = 0$$

$$\frac{da_{3}}{dz} + iK_{31}a_{1} + iK_{32}a_{2} = 0$$
(1)

Assume the input condition of coupler A is shown in (2).

$$a_1(0) = A, a_2(0) = a_3(0) = 0$$
 (2)

 $a_1(0)$, $a_2(0)$, and $a_3(0)$ are complex amplitudes of input optical signals. Reference [6] and [7] give the detail of calculation and showed mathematical formulae for those outputs P₁, P₂, and P₃. Outputs of the coupler are shown in (3)

$$\begin{vmatrix} a_1(L) \end{vmatrix}^2 = A^2 - 2 |a_2(L)^2| |a_2(L)|^2 = |a_3(L)|^2 = \frac{2}{9} A^2 (1 - \cos 3KL)$$
⁽³⁾

Where L is coupling length and $K_{12} = K_{23} = K_{31}$ are assumed to be equal to a constant K since the coupling ratio of 3×3 coupler is 33:33:33. Note that $|a_1(L)|^2$, $|a_2(L)|^2$, and $|a_3(L)|^2$ is optical power and appearing at fiber P₁, P₂, and P₃ respectively.

 P_1 is not used in our design and ignored. Light beams in P_2 and P_3 are then under go phase modulation and launched into the coupler B. They are superimposed inside the coupler B. This superimposed signal is then split based on the properties of the coupler. In order to simplify the calculation, the input conditions of coupler B are modified and shown in (4).

$$a_1(0) = 0, a_2(0) = \frac{B}{\sqrt{2}}e^{i\phi}, a_3(0) = \frac{B}{\sqrt{2}}, B^2 \ge 0$$
 (4)

Where B² is a constant and it is equal to $2 \cdot |a_2(L)|^2$ of coupler A, ϕ is the phase different between $a_2(0)$ and $a_3(0)$. The

term phase difference ϕ is due to phase modulation between coupler A and coupler B. This modulation will be given more explanation in next section.

Substitution (4) into (1) and for position inside coupler z equal to coupling length L, output signals could be calculated and shown in (5)

$$|a_{I}(\phi, L)|^{2} = \frac{2}{9}B^{2}(1 - \cos 3KL)(1 + \cos \phi),$$

$$|a_{II,III}(\phi, L)|^{2} = \frac{1}{18}B^{2}[(7 + 2\cos 3KL))$$
(5)

 $-2\cos\phi(1-\cos 3KL) \mp 6\sin\phi\sin 3KL]$

Note that KL is 30° for coupling ratio equal to 33:33:33. a_{II} is taken the minus sign and a_{III} is taken the plus sign.

B. Relation between phase difference and the vibrating displacement

According to pervious section, signals output are varying with phase difference ϕ . That means the phase difference could be detected by measuring optical power appearing at outputs of 3×3 coupler. It is obvious that the phase difference must be related to the position of vibrating object. The proof of the relationship is shown in following paragraphs.

Considering Fig.1, the laser beam is launch into the coupler through P_1 and outputs are P_1 , P_2 , and P_3 . It is expected that signal power in fiber P_1 , P_2 , and P_3 are as shown in (3). Optical power of P_2 and P_3 are same. Let the optical length of fiber P_2 , P_3 , and collimator is L_2 , L_3 , and L_C respectively. The distance between collimator and vibrating object is D. Distances traveled by signals in P_2 and P_3 before superposition is D_2 and D_3 respectively and equations of D_2 and D_3 are shown in (6)

$$D_{2} = 2L_{2},$$

$$D_{3} = 2L_{3} + 2L_{c} + 2D$$
(6)

Therefore, the path difference is D_3 - D_2 . If it is convert to the phase difference ϕ , the value should be as shown in (7)

$$\phi = \left(D_3 - D_2\right)\frac{2\pi}{\lambda} = \frac{4\pi}{\lambda}\left(L_3 + L_c - L_2\right) + \frac{4\pi D}{\lambda}$$
(7)

 λ is the wavelength of the laser used in this sensor. Since L_c, L₃ and L₂ are fixed values and thus $(4\pi/\lambda) \times (L_3-L_2+L_c)$ is a constant. Only ϕ is a variable varying with the distance between collimator and vibrating object D.

To get a better understanding, the simulation for signal outputs of the interferometer is shown in Fig.3. The simulation is assuming $L_3 + L_c = L_2$ and λ is 1550nm. It is reminded that the power outputs of interferometer are varying with the measuring distance D instead of time.

C. Signal processing

Signals received from the interferometer could be extracted the position information of the vibrating object. However, these signals are not suitable for information extraction using conventional method which is incremental decoding. Some



Fig.4 Flow chart of signal processing

problems are found during information extraction if using conventional method. For example, amplitudes of the optical signals from coupler are not steady since the environmental temperature fluctuation and external vibration will affect the optical power output of the semiconductor laser diode [1]. The phase difference between two optical signals 120° instead of 90° which is used in the conventional decoding. These problems have to be solved in order to measure the displacement.

To solve those problems, signal processing shown in Fig.4 is applied. Reference [1] has already used similar method to generate suitable signals for decoding, which is generating two square waves with 90° phase difference. However, the processing method is very slow because the sampling speed of digital signal processor is too slow to catch up the actual position. Therefore, original signals are distorted and larger error occurs when speed of moving object is higher than 1.2917mm/s.



Fig.5 Compared signals of optical signals from interferometer. Fig.5(a) shows the analog form of compared signals. Fig.5(b) shows the digital form of compared signals

In order to cope with the speed problem, analog circuits are used for handling real-time process and generate high frequency signals for displacement information. There are several functions should be achieved. The detail of analog circuits will be given in section III

A. Envelope Detection

In practical situation, the output power of laser is not exactly a constant because of many reasons such as temperature, external vibration, laser driver control method, etc. [1] The proposed sensor in this paper based on amplitudes of two output signals of interferometer. These output signals are affected by the unstable optical power of laser source.

To solve this problem, upper and lower envelopes are captured by "Envelope Detection function". After envelopes of signals have been received, optical signals will be regulated by other circuits. Digital signal processing will limit the maximum measurable speed and it is not practical in real application situation. Therefore, this envelope detection circuit should be achieved by analog circuit.

B. Offset and Amplitude Calculation

After upper and lower envelopes of optical signal are collected, offsets and amplitudes could be calculated by following equations.

$$C_{off} = \frac{[ue(t) + le(t)]}{2} \tag{8}$$

$$C_{amp} = \frac{[ue(t) - le(t)]}{2} \tag{9}$$

Where C_{off} is the offset of the signal, ue(t) is the upper envelope, le(t) is the lower envelope and C_{amp} is the amplitude of the signal.

The calculation is done by DSP. It is because the variation of the DC offset signal is relatively slow compared with optical signal variation. During experiment, this function is done by DSP with sampling time 0.005s.

C. Normalization

"Normalization function" is adjusting optical signals from interferometer so that offsets are removed and amplitudes are re-amplified. The aim of this function is generating suitable signals which are zero offset and equal amplitude for further processing. The "normalization function" is created and based on the following equation to normalize signals.

$$Normalized_signal = \frac{orginal_signal - C_{off}}{C_{amp}} \quad (10)$$

After signals have passed through this function, output signals were suitable for comparison. Note that this function must be achieved by analog circuit.

D. Comparsion

As mention before, signal outputs of interferometer are not suitable for conventional decoding method even they are normalized. It is because phase difference between two signal outputs is 120° instead of conventional one which is 90° .

There are two solutions for this problem. The first one is creating a new decoding method for 120° phase difference outputs. Another method is modifying 120° phase difference outputs into conventional outputs which are two square waves with 90° phase difference. The latter one is more practical for industrial applications since industries have already adopted incremental encoding for many years.

Reference [1] proposed a comparison method using following conditions to regenerate 4 signals with 45° phase difference

$$condition_1: y + x$$

$$condition_2: y - x$$

$$condition_3: y + 2x$$

$$(11)$$

condition 4:2y + x

x and y stands for normalized signal from P_{II} and P_{III} of interferometer respectively. For digital form signals, if the compared signals are larger than zero, it is defined as "1". It is defined as "0" when compared signals are smaller than zero. The simulations results of compared outputs are shown in Fig.5. [1]

E. Logical operation

In fact, it is possible to using condition_1 and condition_2 to generate incremental signals since compared signals with these two conditions have been already suitable for conventional decoding method. The resolution of this sensor is $\lambda/8 = 191$ nm. However, if XOR gate is applied between condition_1 and condition_2 to generate a signal called phase A and applied between condition_3 and condition_4 to generate a signal called phase B, higher resolution sensor could be produced by using phase A and phase B. New resolution will equal to $\lambda/16 = 95.5$ nm



Fig.6 Experimental setup (upper part in the figure is constructed by optical circuit and lower part is constructed by electric circuit)



Fig.7 Circuit of tran-impedance amplifier

III. EXPERIMENTAL SETUP

The experimental set up is shown in Fig.6. For the signal processing, detail configuration of each circuit is shown in Fig.7, 8, 9, 10, and 11.

All optical components were connected as shown in optical circuit of Fig.6. A 3×3 interferometer together with a 1528nm high coherence laser source (new focus, external-cavity tunable-diode-laser, model number: 6262) which was controlled by laser driver (new focus, inc. external-cavity tunable-diode-laser controller, model number:6200) were used. A mirror was installed on the vibrating target with movable range 1cm. Light reflector was replaced by a fiber reflector proposed by D. B. Mortimore in [8], two PIN photodiode outputs were connected to trans-impedance amplifiers (Fig.7).

Outputs of amplifiers were connected to "envelope detection circuits" (Fig.8). Envelopes information were then coupled to 12 bit A/D converter embedded on dSPACE DS1104 digital signal processing board for real-time measurement. The sampling time of DSP was 0.005s. Inside DSP, the "offset and amplitude calculation" shown in Fig.9 was performed. The calculated values were outputted from D/A converter outputs and connected to "normalization circuits" (Fig.10). Finally, both normalized signals were compared using 'comparison circuit" (Fig.11). Four compared



Fig.8 Circuit for envelope detector



Fig.9 Algorithm for offset and amplitude calculation

signals with condition shown in (11) were connected to XOR gates and generated phase A and B.

An optical encoder with 50nm resolution (Renishaw, linear optical encoder, RGH24H30D30A) was also installed for reference displacement measurement and it was connected to dSPACE board. Also, phase A and phase B of proposed sensor were also connected to dSPACE board for displacement conversion. Environmental temperature is 22°C and no temperature control for electric circuits.

IV. EXPERIMENTAL RESULT AND DISCUSSION

Result is shown in Fig. 12. Fig.12(a) shows positions of the vibrating target measured by reference sensor and proposed sensor. Fig.12(b) shows the error between displacement measured by proposed sensor and reference senor. Fig.12(c) shows the velocity of the vibrating object. Note that Fig12(a) has two lines but they are overlapped with each other. It is because the error is too small to distinguish. For Fig.12(c), the velocity was calculated by reference displacement. Note that the velocity is obtained by offline calculation.

Some phenomena are observed during this experiment. By observing Fig.12(b), average error is not zero because zero marks of reference sensor and proposed sensor were not exactly equal. The misalignment in our case is about 22 µm which shown on Fig.12(b). The error maximum deviation which is located at around 2.25s is about 1µm. Another phenomenon shows on Fig.12(b) is related to the trend of the error curve. The error curve is gradually decreasing from time Os to 2s. It is then increasing from time 2s to 5s. In fact, the reason is still investigated and it may be resulted from small temperature variation in environment and overheat in electric components. This assumption is because the frequency of trend variation is very low. This frequency should not be created by any electric circuits or any mechanism vibration and the variation period is longer than 10s and the variation is not in regular manner.

The measurable velocity of proposed sensor with analog circuits is much higher than sensor using digital signal processor (DSP) for signal processing [1]. According to [1], maximum measurable velocity of sensor using DSP is 1.2917mm/s. The limitation is caused by the DSP sampling speed. Low sampling speed distorted the original analog signal. In proposed design, all fast signals are handled by analog circuits and thus signals are not distorted. The measurable



Fig.10 Circuit for normalization function



speed limitation is caused by the cutoff frequency of electric circuits. In our setup, the cutoff frequency of all electric circuit is 100 kHz. Optical interferometer generates one cycle of signal when the target moved $\lambda/2$. Therefore, the maximum velocity which could be handled by the sensor is shown in (12)

$$V_{\rm max} = f_{cutoff} \times \lambda / 2 \tag{12}$$

where V_{max} is the maximum measurable velocity, f_{cutoff} is minimum cutoff frequency of the envelope detector circuit and λ is the wavelength of laser.

In theory, the sensor should be able to detect displacement of the vibrating object if the speed is less than 77mm/s and this is much powerful than the sensor shown in [1].

V. CONCLUSION

This paper proposed a practical displacement sensor, after some modifications related to the trend of error, with using interesting idea which is integrating the optical 3×3 optical coupler, narrow spectrum laser source, and electrical circuits to form an interferometric displacement sensor. Advantages of this sensor are high precision displacement measurement and with low production cost. It is suitable for differential displacement measurement, such as vibrating diaphragm on machine and continuous vibrating mechanism.

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REFERENCES



Fig.12 Results of the experiment. Fig.12(a) Position-time graphs measured by proposed sensor and reference sensor. Fig.12(b) Error-time graph between proposed sensor and reference sensor measurements. Fig.12(c) Velocity-time graph of the vibrating target.

- H.W.Chow, N.Cheung, W.Jin, "A Low Cost Sub-Micro Linear Incremental Encoder Based on 3×3 Fiber Optic Directional Coupler", IEEE Transactions on instrumentation and measurement, unpublished.
- [2] G. Lai, T. Sato, M. Wang, S. Shinohara, Y. Wu, H. Ikeda, "Laser-Diode Distance and Displacement Sensor with Double External Cavities", Industry Applications Conference, 2000. Conference Record of the 2000 IEEE, Volume 2, 8-12 Oct. 2000 Page(s):1024 - 1028 vol.2
- [3] B.P.Konkel, E.C.Zipter, E.E.Kirkham, "Improved parallelism measurement using a laser interferometer", Industry Applications Society Annual Meeting, 1988., Conference Record of the 1988 IEEE, 2-7 Oct. 1988 Page(s):1387 - 1390 vol.2
- [4] Z. Ding; Y. Nagaike,; G. Lai, "Fiber-optic based scanning confocal microscopic interferometer for the measurement of surface topography in manufacturing process", Industry Applications Conference, 2000. Conference Record of the 2000 IEEE, Volume 2, 8-12 Oct. 2000 Page(s):1029 - 1032 vol.2
- [5] M.C. Tomic, "Low-coherence interferometric method for measurement of displacement based on a 3 3 fiber-optic directional coupler," J. Opt. A: Pure Appl. Opt., vol.4 Nov.2002
- [6] S. K. Sheem, "Fiber-optic gyroscope with (3×3) directional coupler," Appl .Phys. Lett., vol. 37. pp869-871, Nov.1980
- [7] S. K. Sheem, "Optical fiber interferometers with 3×3 directional couplers: analysis," J.Appl .Phys., vol. 52. pp3865-3872, Jun.1981
- [8] S. Donati, L. Falzoni, and S. Merlo, "A PC-interfaced, compact laserdiode feedback interferometer for displacement measurements," IEEE Trans. Instrum. Meas., vol. 45, pp. 942–947, Dec. 1996.