Evaluation of Torque Sharing Functions for Torque Ripple Minimization of Switched Reluctance Motor Drives in Electric Vehicles

X. D. Xue, K. W. E. Cheng, and N. C. Cheung
Department of Electrical Engineering, the Hong Kong Polytechnic University
Hung Hom, Kowloon
Hong Kong, China

***Abstract*-Generic algorithm is used to optimize the turn-on angle and overlap angle and the torque ripple factor is selected as the fitness function. Four types of torque sharing functions for switched reluctance motor drives are evaluated by using the torque ripple factor. The investigation shows that the cubic torque sharing function is the prime choice in consideration of the minimum torque ripple, the simplicity and speediness of real-time computation. Therefore, this paper provides a valuable guide for selecting the appropriate torque sharing function to implement torque ripple minimization in switched reluctance motor drives.**

I. INTRODUCTION

Due to simple and low-cost construction, low weight, easy cooling, excellent torque-speed characteristics, high torque, high operating efficiency, and inherent fault tolerance, switched reluctance motor (SRM) drives are prime choice for electric motor drives in EVs. However, torque ripple is an inherent drawback of SRM drives. Hence, control of torque ripple minimization is a challenging study area [1]. Torque sharing function (TSF) is an effective approach to implement torque ripple minimization in SRM drives. Some torque sharing functions were developed to successfully accomplish torque ripple minimization [1] – [5]. Therefore, evaluation and comparison of those torque sharing functions are very helpful to select an appropriate torque sharing function, in order to reduce torque ripple in SRM drives.

In this paper, the torque ripple factor (TRF) is defined to evaluate torque ripple minimization implemented by torque sharing functions. Four types of torque sharing functions, which are linear, cubic, sinusoidal and exponential functions, are discussed. In order to compare torque sharing functions for minimum torque ripple, generic algorithm (GA) is used to optimize torque sharing functions and the torque ripple factor is selected as the fitness function. The investigation shows that the cubic, sinusoidal and exponential torque sharing functions have the similar fitness values, which are considerable smaller than the one from the linear torque sharing function. Taking into account the minimum torque ripple, the simplicity and speediness of real-time computation, the cubic torque sharing function is the prime choice for torque ripple minimization in SRM drives.

II. TORQUE SHARING FUNCTIONS

The torque sharing function in a rotor period is defined as

$$F_{TSF}(\theta) = \begin{cases} 0, & (0 \le \theta \le \theta_{on}) \\ p_{up}(\theta), & (\theta_{on} \le \theta \le \theta_{on} + \theta_{ov}) \\ T_d, & (\theta_{on} + \theta_{ov} \le \theta \le \theta_{off}) \\ p_{dn}(\theta), & (\theta_{off} \le \theta \le \theta_{off} + \theta_{ov}) \\ 0, & (\theta_{off} + \theta_{ov} \le \theta \le \theta_p) \end{cases} \quad (1)$$

where θ_{on} denotes the turn-on angle, θ_{ov} denotes the overlap angle, θ_{off} denotes the turn-off angle, T_d denotes the desired torque produced by the SRM, θ denotes the rotor position, θ_p denotes the rotor period, $p_{up}(\theta)$ and $p_{dn}(\theta)$ denotes the rising portion and declining portion of torque sharing functions, respectively.

Fig 1 illustrates the typical profile of the torque sharing function.

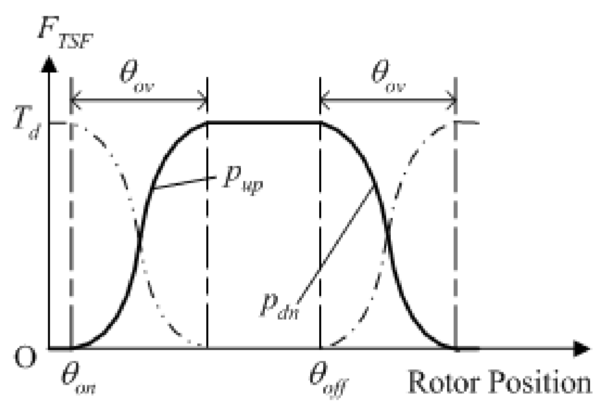

Fig. 1. Typical profile of torque sharing functions

A. Linear Torque Sharing Function

The linear torque sharing function means that the torque produced by two phases changes with the rotor position linearly, during phase commutation. A linear torque sharing

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function was presented in [2], where the linear torque change with the rotor position is required during the overlap of two phases.

The rising portion $p_{up}^{l}(\theta)$ is proposed as

$$p_{up}^{l}(\theta) = \frac{T_d}{\theta_{ov}}(\theta - \theta_{on}) \tag{2}$$

Consequently, the declining portion $p_{dn}^{l}(\theta)$ can be expressed as

$$p_{dn}^{l}(\theta) = T_d - \frac{T_d}{\theta_{ov}}(\theta - \theta_{off}) \tag{3}$$

The phase corresponding to $p_{up}^{l}(\theta)$ is named as the in-coming phase and the phase corresponding to $p_{dn}^{l}(\theta)$ is named as the out-going phase. Thus, the in-coming phase and the out-going phase are active simultaneously during phase commutation and only an active phase occurs during phase non-commutation.

B. *Cubic Torque Sharing Function*

The cubic torque sharing function indicates that the torque produced by the phases, during phase commutation, changes nonlinearly with the rotor position. This nonlinearity has the form of the cubic polynomial. The cubic torque sharing function was developed in [3]. The rising portion $p_{up}^{c}(\theta)$ is defined as

$$p_{up}^{c}(\theta) = \frac{3T_d}{\theta_{ov}^2}(\theta - \theta_{on})^2 - \frac{2T_d}{\theta_{ov}^3}(\theta - \theta_{on})^3 \tag{4}$$

Thus, the computation of the declining portion $p_{dn}^{c}(\theta)$ is given by

$$p_{dn}^{c}(\theta) = T_d - p_{up}^{c}(\theta - \theta_{off} + \theta_{on}) \tag{5}$$

C. *Sinusoidal Torque Sharing Function*

The sinusoidal torque sharing function means that the torque produced by the phases, during phase commutation, changes with the rotor position in terms of the sinusoidal function. On the basis of the sinusoidal torque sharing function presented in [4], in this paper, the improved sinusoidal torque sharing function is developed. The rising portion $p_{up}^{s}(\theta)$ is defined as

$$p_{up}^{s}(\theta) = \frac{T_d}{2} - \frac{T_d}{2}\cos\frac{\pi}{\theta_{ov}}(\theta - \theta_{on}) \tag{6}$$

Hence, the declining portion $p_{dn}^{s}(\theta)$ is computed from

$$p_{dn}^{s}(\theta) = \frac{T_d}{2} + \frac{T_d}{2}\cos\frac{\pi}{\theta_{ov}}(\theta - \theta_{off}) \tag{7}$$

D. *Exponential Torque Sharing Function*

Similarly, the exponential torque sharing function means that the torque produced by the phases, during phase commutation, changes with rotor position in terms of the exponential function. In [5], an exponential torque sharing function was presented. For that exponential torque sharing function, the rising portion and the declining portion are not symmetrical about the middle position. Furthermore, it only depends on the turn-on angle. In this paper, an improved exponential torque sharing function is proposed. The rising portion $p_{up}^{e}(\theta)$ is expressed as

$$p_{up}^{e}(\theta) = T_d[1 - \exp(-(\theta - \theta_{on})^2 / \theta_{ov})] \tag{8}$$

Thus, the declining portion $p_{dn}^{e}(\theta)$ is given as

$$p_{dn}^{e}(\theta) = T_d \exp(-(\theta - \theta_{off})^2 / \theta_{ov})] \tag{9}$$

E. *Typical Profiles of Torque Sharing Functions*

Fig. 2 shows the typical profiles of the above four torque sharing functions. From top to bottom, the profiles in the Fig. 2 are the cubic torque sharing function, exponential torque sharing function, linear torque sharing function, and sinusoidal torque sharing function, in turn. The desired torque is 5 Nm. The turn-on angle is 5 degree and the overlap angle is 6 degree.

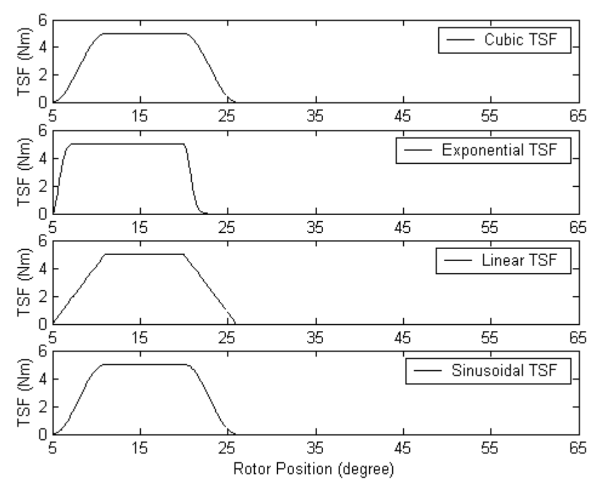


Fig. 2. Typical profiles of four torque sharing functions

III. Control Scheme of Torque Ripple Minimization

The block diagram of the proposed control scheme to implement torque ripple minimization is shown in Fig. 3. The

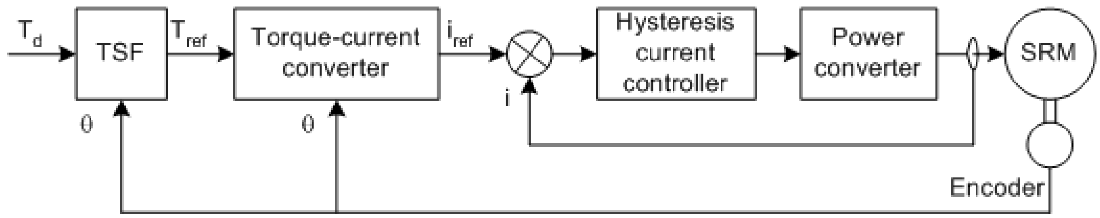


Fig. 3. Block diagram of the proposed control scheme

proposed control scheme can be described as follows. Firstly, the torque reference is determined from the desired torque, the rotor position, and the torque sharing function. Next, the torque-current converter is used to compute the current reference from the torque reference and the rotor position. Then, the current reference is compared with the measured current. After that, the error signal is input to the hysteresis current controller. Finally, the PWM signal generated by the hysteresis current controller is utilized to drive the power converter of the SRM.

In order to accurately track the current reference, the developed hysteresis current controller respects the two rules. One of which is that the soft-chopping is adopted when the rotor position is not smaller than the turn-on angle and smaller than the turn-off angle. The other is that the hard-chopping is adopted when the rotor position is not smaller than the turn-off angle.

IV. Criterion of Torque Ripple Minimization

A. *Torque Ripple Factor*

In order to evaluate effects of torque sharing functions on torque ripple minimization, a criterion is needed to be defined as the performance index of torque ripple minimization. In this paper, the torque ripple factor is proposed as the criterion of torque ripple minimization, and is defined as

$$TRF = \frac{T_{\max} - T_{\min}}{T_{ave}} \tag{10}$$

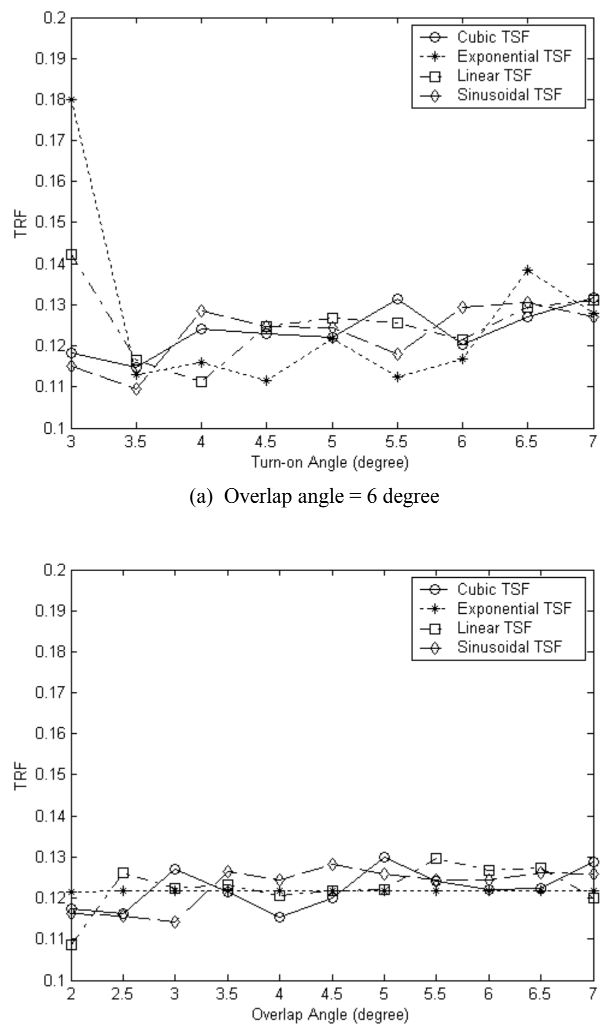
where $T_{\max}$ represents the maximum torque, $T_{\min}$ represents the minimum torque, and the T_{ave} represents the average torque.

It can be seen from (10) that the small torque ripple factor means the small torque ripple and that the large torque ripple factor does the large torque ripple. Thus, the torque ripple factor is suitable for evaluating torque ripple in SRM drives.

B. *Torque Ripple Factor Function*

Clearly, the torque ripple factor depends on maximum torque, minimum torque and average torque. For the specific SRM drive, hence, the torque ripple factor depends on the turn-on angle, turn-off angle, motor speed and desired torque. Consequently, the torque ripple factor is dependent on the turn-on angle and the overlap angle for the given speed and torque since the turn-off angle can be computed from the turn-on and overlap angles.

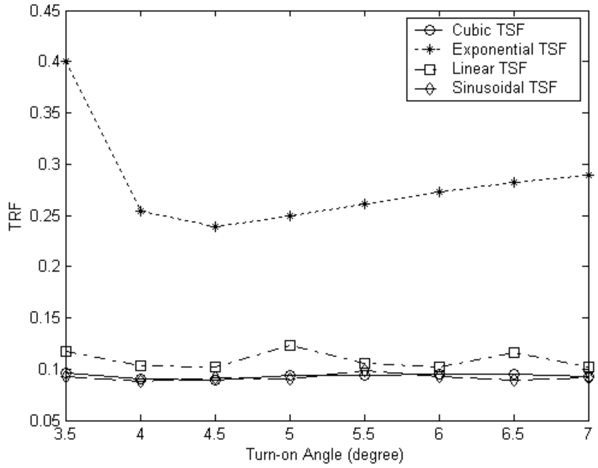
A four-phase SRM drive is utilized to investigate the torque ripple factor. When the SRM drive operates at the motor speed of 200 rpm and the desired torque of 2 Nm, and the motor



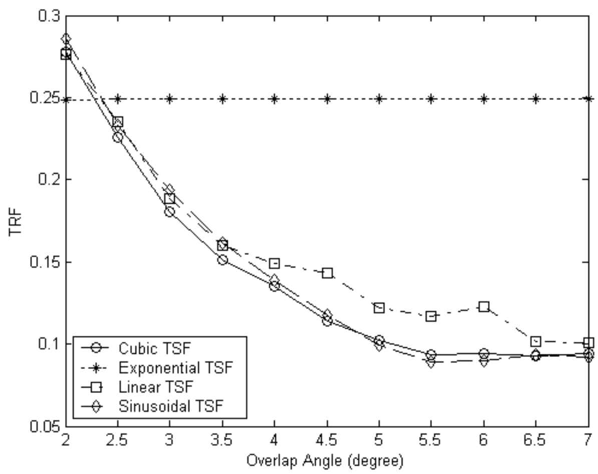
(a) Overlap angle = 6 degree

(b) Turn-on angle = 5 degree

Fig. 4 Change of torque ripple factor (speed = 200 rpm and desired torque = 2 Nm)

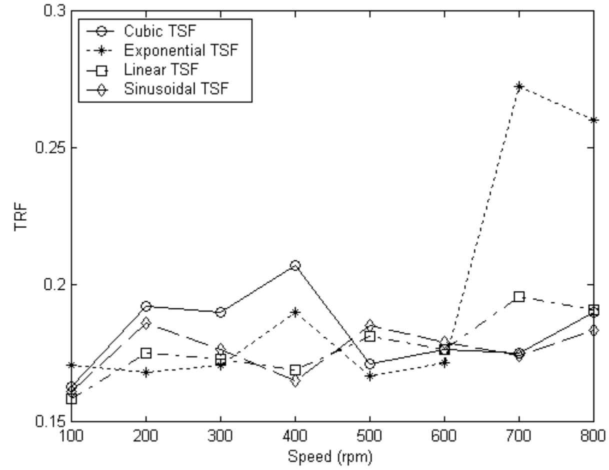


(a) Overlap angle = 6 degree

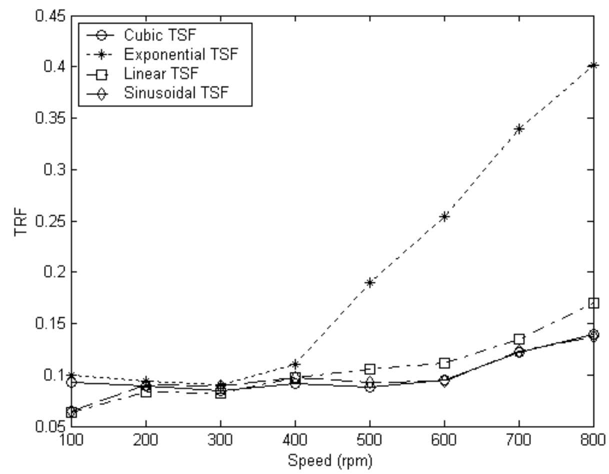


(b) Turn-on angle = 5 degree

Fig. 5 Change of torque ripple factor (speed = 600 rpm and desired torque = 3 Nm)



(a) Desired torque = 1 Nm



(b) Desired torque = 3 Nm

Fig. 6 TRF comparisons between four TSFs with the specified turn-on and overlap angles

speed of 600 rpm and the desired torque of 3 Nm, the changes of the torque ripple factor with the turn-on angle and the overlap angle are depicted in Fig. 4 and Fig. 5, respectively.

It can be observed from Fig. 4 and Fig. 5 that for the given overlap angle the turn-on angle has the significant effect on the torque ripple factor if the exponential torque sharing function is used and has the weak effect on the torque ripple factor if the cubic, linear or sinusoidal torque sharing function is used. On the other hand, for the given turn-on angle, the overlap angle has no effect on the torque ripple factor if the exponential torque sharing function is employed and has the considerable effect on the torque ripple factor if the cubic, linear or sinusoidal torque sharing function is employed. For the given speed and desired torque, hence, the torque ripple factor depends on the turn-on and overlap angles. In order words, the torque ripple factor is dependent on the torque sharing function. For anyone of four torque sharing functions, furthermore, the turn-on angle and the overlap angle can be optimized for minimizing the torque ripple factor at the given speed and desired torque.

Fig. 6 shows the changes of torque ripple factor with the motor speed when the turn-on angle is selected as 4 degree and the overlap angle is selected as 5 degree. In this case, it can be observed that the sinusoidal, cubic or linear torque sharing function results in the small torque ripple factor and the exponential torque sharing function leads to the larger torque ripple factor than the other three torque sharing functions.

V. Optimization of Torque Sharing Functions

A. Optimization Method

From the analysis in the last section, it can be found that the turn-on angle, overlap angle, speed and desired torque have effects on the torque ripple factor. Furthermore, torque sharing functions at the given motor speed and torque depend on the turn-on angle and overlap angle. In other words, a torque sharing function will result in the various torque ripple factors due to the different turn-on and overlap angles. For four torque sharing functions at the specified speed and torque, however, the minimum torque ripple factor can be found if the turn-on angle and overlap angle are optimized. In this paper, thus, the torque sharing functions to lead to the minimum torque ripple factors are evaluated and compared.

Due to the nonlinear SRM model, the minimization of torque ripple factor is a nonlinear optimization problem with multi-variables. In this paper, consequently, generic algorithm (GA) is selected as the solution to such an optimization problem. In the meanwhile, it can be found that four torque sharing functions cannot result in the minimum torque ripple factors simultaneously for the same turn-on angle, overlap angle, speed or desired torque. In order to evaluate and compare four types of torque sharing functions, the following approach is selected: (i) for specific torque and speed, four types of torque sharing functions are optimized to minimize torque ripple factor; (ii) for specific torque, the average minimum torque ripple factor is computed at various specific speed, and (iii) four types of torque sharing functions are compared and evaluated from the average minimum torque ripple factor values at the specific torque.

B. *Fitness Function*

Based on the discussion in the last section, at the specified speed and desired torque, the torque ripple factor is the function of the turn-on angle and the overlap angle. Hence, it can be expressed as

$$TRF = f(\theta_{on}, \theta_{ov}) \tag{11}$$

Hence, the optimized variables can be selected as the turn-on angle and the overlap angle. Obviously, the optimization objective is the torque ripple factor. The fitness function can be defined as

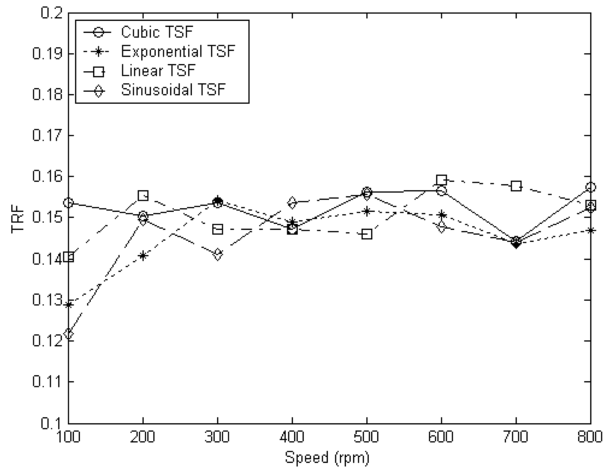
$$FitnessFunction = \min(TRF) \tag{12}$$

C. *Optimization Results*

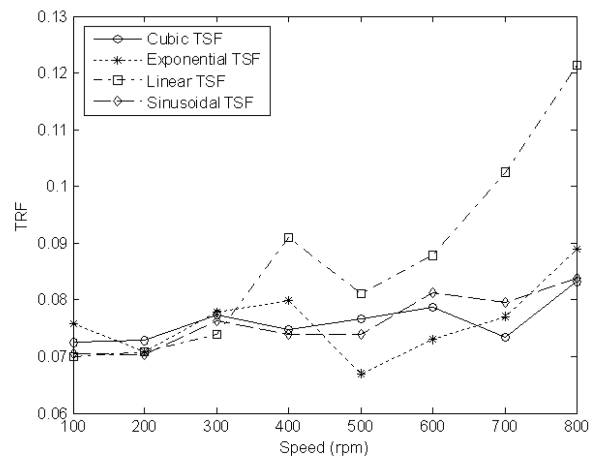
Fig. 7 illustrates the fitness values of torque ripple factor for four torque sharing functions when the turn-on and overlap angles are optimized by using GA.

D. *Evaluation of Four Types of Torque Sharing Functions*

Comparing Fig. 7 with the figures shown in Fig. 4 to Fig. 6, it can be seen that the fitness values of torque ripple factor from the optimized turn-on and overlap angles are much smaller than the torque ripple factor values from the turn-on and overlap angles without optimization, for anyone of four torque sharing functions. Hence, the optimization of torque



(a) Desired torque = 1 Nm



(b) Desired torque = 3 Nm

Fig. 7 Fitness values of TRF computed by using GA

sharing functions is beneficial to fulfilling the best torque ripple minimization.

Table I indicates the average fitness values generated by the cubic, exponential, sinusoidal and linear torque sharing functions at the specific torque. It can be observed from Fig. 7 and Table I that the fitness values of torque ripple factor are the approximately same for the cubic, exponential and sinusoidal torque sharing functions and that the fitness values of torque ripple factor generated by the linear torque sharing function are larger than the ones generated by the other three torque sharing functions. Thus, the fitness values of torque ripple factor generated by the nonlinear torque sharing function are smaller than the ones generated by the linear torque sharing function. As for the torque ripple factor in SRM drives, consequently, the cubic, exponential and sinusoidal torque sharing functions are better than the linear torque sharing function.

TABLE I

AVERAGE FITNESS VALUES OF TSFS

Torque (Nm)	Average fitness value			
	Cubic TSF	Exponential TSF	Sinusoidal TSF	Linear TSF
1	0.146448	0.145729	0.145775	0.150802
3	0.076199	0.076282	0.076188	0.087361

In addition, comparing three nonlinear torque sharing functions, the cubic torque sharing function has the simplest mathematical expression. Thus, it also has the quickest computation among three nonlinear torque sharing functions.

VI. CONCLUSIONS

Four types of torque sharing functions to implement torque ripple minimization in SRM drives have been discussed. For the specified torque, torque sharing functions depend on the turn-on angle and the overlap angle. The study in this paper has indicated that torque sharing functions with different turn-on and overlap angles have considerable effects on the torque ripple factor. Consequently, through optimizing torque sharing functions at the specific torque and motor speed, the minimum torque ripple can be accomplished under the proposed control scheme.

The simulation results show that the cubic, sinusoidal, and exponential torque sharing functions result in the similar minimum torque ripple factors, which are much smaller than the torque ripple factor produced by the linear torque sharing function. Hence, these three torque sharing functions are good selection for torque ripple minimization. However, the cubic torque sharing function has the simplest expression and the quickest computation at real-time. Considering the minimum torque ripple, the simplicity and speediness of real-time computation, therefore, the cubic torque sharing function is the prime choice. In summary, this paper provides a valuable guideline for selecting a good torque sharing function to implement torque ripple minimization in SRM drives. It is beneficial to improve performances of SRM drives in EVs.

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