# PASSIVITY-BASED CONTROL FOR SPEED REGULATION IN PERMANENT-MAGNET LINEAR MOTORS

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**Abstract:** A speed regulation method with passivitybased control (PBC) for a linear direct-drive AC servo system is presented. The friction and force ripple are effectively minimized by designing a feedback controller with adaptability. Simulation verifies that the proposed control scheme not only suppresses the friction and force ripple problem of permanent-magnet linear synchronous motor (LPMSM), but also makes the direct-drive system with strong robustness to parameter variations and external disturbances.

Key words: passivity-based control, linear AC servo system, adaptive control

## **1. INTRODUCTION**

The PMLM makes use of electromagnetic energy to generate linear motion directly. The increasingly widespread industrial applications of PMLM in various semiconductor processes, precision metrology, and miniature system assembly are self-evident testimonies of the effectiveness of PMLM in addressing the high requirements associated with these application areas. The main benefits of a PMLM include the high force density achievable, low thermal losses and high precision and accuracy associated with the simplicity in mechanical structure. Linear motor requires no indirect coupling mechanisms such as gear boxes, chains, and screws coupling. So, it is possible to make PMLM and its control devices fabricated into a unity. However, the advantages associated with mechanical transmission are also consequently lost, such as the inherent ability to reduce the effects of model uncertainties and external disturbances. The perturbations of parameters, disturbance of external load, and, especially the inherent end effect and cogging effect will damage the precision and accuracy. It is necessary to reduce these effects, either through proper physical design or via the control algorithm, to achieve high-speed and high precision.

The more predominant nonlinear effects underlying a linear motor system are the friction and force ripples (detent and reluctance forces). Due to nonlinear characteristics of friction and ripple force, the control of this system is particularly challenging since conventional PID control does Norbert C. Cheung Department of electrical engineering Hong Kong Polytechnic University, Hong Kong

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not suffice in the high precision and high speed applications. Several control strategies are applied to overcome these problems. In [1], an adaptive sliding mode control scheme is applied to identify the unknown disturbances and a superior tracking performance compared to conventional PID control is achieved. In [2], a sliding mode variable structure control method based on thrust-observer is presented. The chattering arising by sliding mode control and ripple force is effectively minimized by designing a load thrust observer and disturbance feed forward compensation. In [3],  $H_{\infty}$  control strategy is applied to suppress the adverse influences of friction and ripple force on the PMLM.

In view of the time-varying nonlinear characteristics and the uncertainties of modeling to PMLM, the robustness of control algorithm and its availability are predominant for the dynamics of PMLM depending on operation condition. In this paper, we apply passivity-based control (PBC) strategy to suppress the friction and ripple force phenomena. The PBC is designed by reshaping the system's natural energy and injecting the required damping to achieve the control objective <sup>[4]</sup>. The attractive features of this approach are the enhanced robustness and lack of (controller calculation) singularities, properties which stem from the fact that cancellation of system nonlinear terms is avoided. We also apply the adaptive method to identify the time-varying parameters to compensate the uncertainties of the model and the outside perturbations. This control scheme guarantees the stability globally and the convergence of the state errors and the errors in parameter estimate exponentially.

#### 2. MODEL OF PMLM

The dynamics of PMLM can be described as Park equation in the coordinate synchronously rotating with rotor.

$$\begin{split} L_{d}\dot{i}_{d} &= -R_{s}i_{d} + k_{1}L_{q}i_{q}V_{m} + u_{d} \\ L_{q}\dot{i}_{q} &= -R_{s}i_{q} - k_{1}(L_{d}i_{d} + \lambda_{af})V_{m} + u_{q} \\ M\dot{V}_{m} &= k_{2}[\lambda_{af}i_{q} + (L_{d} - L_{q})i_{d}i_{q}] - F \end{split}$$
(1)

Where

$$u_d$$
,  $u_q$  the d-axis and q-axis winding voltage;

$$i_d$$
,  $i_q$  the d-axis and q-axis winding current;

 $R_s$  winding resistance;

$$L_d$$
,  $L_q$  the d-axis and q-axis winding inductance;

 $\lambda_{af}$  flux of permanent magnet;

 $V_m$  speed of mover;

- $\tau$  pitch;
- *P* number of poles;
- *M* moving thrust block mass;
- *F* system disturbance.

$$k_1 = \frac{\pi}{\tau}, \quad k_2 = \frac{3}{2} \frac{P\pi}{\tau}$$

 $F = f_f + f_r + f_l$ .  $f_f$  is friction force and maybe written as<sup>[1]</sup>:

$$f_f = [f_c + (f_s - f_c)e^{-(V_m/v_s)^2} + f_v V_m]sign(V_m)$$

Where  $f_s$  is static friction.  $f_c$  is Coulomb friction.  $f_v$  is viscous friction coefficient.  $v_s$  is lubricant coefficient. Fig.2 graphically illustrates a basic friction model.



Fig.2 Curve of nonlinear friction

The load force  $f_i$  is assumed to be bounded within the unknown upper bound as follows:

 $\left|f_{l}(t)\right| < f_{lm} \quad \forall t > 0.$ 

The force ripple arising from the cogging and reluctance forces is a periodic function of the displacement of mover and is usually described by a sinusoidal function of the load position with a period of  $\omega$  and an amplitude of  $A_r$ :

$$f_r = A_r \sin(\alpha s + \varphi) = A_{r_1} \cos(\alpha s) + A_{r_2} \sin(\alpha s), \quad s = \int_0^t V_m dt$$

The state vector is selected as:  $x = D[i_d, i_q, V_m]^T$ 

Where 
$$D = diag \left\{ L_d, L_q, \frac{2}{3} \frac{M}{P} \right\}$$

The positive definite energy function is defined as:

$$H(x) = \frac{1}{2} \left( L_d i_d^2 + L_q i_q^2 + \frac{2}{3} \frac{M}{P} V_m^2 \right)$$

With this energy function, the model (1) of PMLM can be rearranged as the formula of port-controlled Hamiltonian (PCH):

$$\dot{x} = \left[J(x) - R\right] \cdot \frac{\partial H}{\partial x}(x) + g \cdot \mathbf{u} + \xi$$
(2)

Where

$$g = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \ u = \begin{bmatrix} u_d \\ u_q \end{bmatrix}, \ R = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & B \end{bmatrix}, \ \xi = \begin{bmatrix} 0 \\ 0 \\ \xi_3 \end{bmatrix}$$
$$\xi_3 = -\frac{2}{3P} [f_c + (f_s - f_c)e^{-(V_m/v_3)^2} + A_r \sin(\omega s + \varphi) + f_l]$$
$$J(x) = \begin{bmatrix} 0 & 0 & k_1 x_2 \\ 0 & 0 & -k_1 (x_1 + \lambda_{af}) \\ -k_1 x_2 & k_1 (x_1 + \lambda_{af}) & 0 \end{bmatrix}$$

# **3. THE DESIGN OF PBC FOR THE SPEED REGULATION OF PMLM**

Defining state error as:  $e=x-x_d$ .  $x_d$  is the reference state vector, which defines the expected performance of system. Substituting  $e=x-x_d$  into equation (2) and yielding the model of the state error as:

$$\dot{e} + [R - J(x)] \cdot D^{-1} \cdot e = \Phi \tag{3}$$

Where  $\Phi = g \cdot u + \xi - \dot{x}_d - [R - J(x)] \cdot D^{-1} \cdot x_d$ 

Defining the energy function for the dynamics of the state error (3) as:  $V = \frac{1}{2}e^{T}e^{T}$ 

Differentiate the energy function V along with the

trajectory of dynamics (3) and get

$$\dot{V} = -e^T R D^{-1} e + e^T J(x) D^{-1} e + e^T \Phi$$

The mapping from  $\Phi$  to e is passivity <sup>[4]</sup>. The passivity system is asymptotically stable with no outside force acting on it. J(x) is antisymmetric matrix. So  $e^T J(x)e = 0$ . The term of  $-J(x)D^{-1}e$  imposes no influence on the stability to the dynamics of state error and is called workless force. R is positive definite matrix. Making the  $\Phi$  to zero by selecting suitable control law u, we conclude the result  $\dot{V} = -e^T R D^{-1}e$ . The state error system is asymptotically stable. Furthermore, the state error is convergent exponentially, i.e.  $||e|| < k_0 e^{-\mu_0} ||e(0)||$ , where  $k_0$  and  $\mu_0$  are positive scalars. The rate of convergence can be improved by injecting damp <sup>[6]</sup>.

The reference state  $x_d$  is similar to the reference model in the reference model adaptive control. We take the  $x_d$  as following form:

$$x_{d1} = 0$$

$$x_{d2} = \frac{L_q}{k_1 \lambda_{af}} \left( \frac{2}{3P} BV_* + f_c + f_l \right)$$

$$\dot{x}_{d3} + kx_{d3} - \frac{2}{3} \frac{kMV_*}{P} = 0, \ x_{d3}(0) = x_{d3}^0$$
(4)

Where  $V_*$  is the reference speed Constructing the feedback control law as:

$$u_{d} = -\frac{k_{1}}{L_{d}} x_{2} x_{d3}$$

$$u_{q} = \dot{x}_{d2} + \frac{R}{L_{q}} x_{d2} + k_{1} (x_{1} + \lambda_{af}) x_{d3}$$
(5)

By the  $x_d$  as (4) and control scheme as (5),  $\Phi$  will converge to zero asymptotically. So  $e \to 0$ , i.e.  $x \to x_d$ . Finally, the state will achieve the stable equilibrium:

$$x_{*} = \begin{bmatrix} x_{1*} \\ x_{2*} \\ x_{3*} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{k_{1}\lambda_{af}} \begin{bmatrix} \frac{2M}{3P} BV_{*} + f_{l} + f_{c} \end{bmatrix} \\ \frac{2}{3} \frac{M}{P} V_{*} \end{bmatrix}.$$

#### 4. PBC WITH ADAPTIVE ESTIMATE

The friction, load and reluctance are uncertain. In order

to eliminate the adverse influence of original errors to PBC, the adaptive technique is applied to identify the unknown parameters. With these estimate of parameters, the feedback control law is constructed.

Replace the actual values of R,  $f_c$ ,  $f_v$ , f,  $A_{r1}$ ,  $A_{r2}$ by their estimate  $\hat{R}$ ,  $\hat{f}_c$ ,  $\hat{f}_v$ ,  $\hat{f}$ ,  $\hat{A}_{r1}$ ,  $\hat{A}_{r2}$ .

Where 
$$\widetilde{R} = R - \hat{R}$$
,  $\widetilde{f}_c = f_c - \hat{f}_c$ ,  $\widetilde{f}_v = f_v - \hat{f}_v$ ,  $\widetilde{A}_{r1} = A_{r1} - \hat{A}_{r1}$ ,  
 $\widetilde{A}_{r2} = A_{r2} - \hat{A}_{r2}$ ,  $\widetilde{f} = f - \hat{f}$ ,  $f = f_{fM} + f_{lM}$ .

Take the energy function of uncertainty system as:

$$V_{A} = \frac{1}{2}e^{T}e + \frac{1}{2L_{q}g_{1}}\widetilde{R}^{2} + \frac{1}{2g_{2}}\widetilde{f}_{c}^{2} + \frac{1}{2g_{3}}\widetilde{f}^{2} + \frac{1}{2g_{4}}\widetilde{f}_{v}^{2} + \frac{1}{2g_{4}}\widetilde{f}_{v}^{2} + \frac{1}{2g_{5}}\widetilde{A}_{r1}^{2} + \frac{1}{2g_{6}}\widetilde{A}_{r2}^{2}$$
(6)

Differentiate the energy function  $V_A$  along with the trajectory of dynamics (3)

$$\dot{V}_{A} = -e^{T} \operatorname{Re} + e^{T} J(x)e + \frac{1}{L_{q}} x_{d2} e_{2} \widetilde{R} + \frac{3P}{2M} x_{d3} \widetilde{f}_{v} + e_{3} \widetilde{f}_{c}$$

$$+ e_{3} \widetilde{f} + e_{3} \widetilde{f}_{v} + e_{3} \widetilde{A}_{r1} + e_{3} \widetilde{A}_{r2} + (\widetilde{f}_{s} - \widetilde{f}_{c}) e^{-(\frac{v}{v_{s}})^{2}}$$

$$+ \frac{1}{L_{q} g_{1}} \widetilde{R} \dot{\widetilde{R}} + \frac{1}{g_{2}} \widetilde{f}_{c} \dot{\widetilde{f}}_{c} + \frac{1}{g_{3}} \widetilde{f} \dot{\widetilde{f}} + \frac{1}{g_{4}} \widetilde{f}_{v} \dot{\widetilde{f}}_{v}$$

$$+ \frac{1}{g_{5}} \widetilde{A}_{r1} \dot{\widetilde{A}}_{s} + \frac{1}{g_{6}} \widetilde{A}_{r2} \dot{\widetilde{A}}_{r2}$$

$$\leq -e^{T} \operatorname{Re} + e^{T} J(x)e + \frac{1}{L_{q} g_{1}} (\dot{\widetilde{R}} + g_{1} x_{d2} e_{2}) \widetilde{R}$$

$$+ \frac{1}{g_{2}} (\dot{\widetilde{f}}_{c} + g_{2} e_{2}) \widetilde{f}_{c} + \frac{1}{g_{3}} (\dot{\widetilde{f}} + g_{3} e_{2}) \widetilde{f} + \frac{1}{g_{4}} (\dot{\widetilde{f}}_{v} + g_{3} e_{2}) \widetilde{f}_{v}$$

$$+ \frac{1}{g_{5}} (\dot{\widetilde{A}}_{r1} + g_{5} e_{3}) \widetilde{A}_{r1} + \frac{1}{g_{6}} (\dot{\widetilde{A}}_{r2} + g_{6} e_{3}) \widetilde{A}_{r2}$$

$$(7)$$

Design the adaptive laws for the

$$\tilde{\vec{R}} = -g_1 x_{d2} e_2 \tag{8}$$

$$\tilde{f}_c = -g_2 e_2 \tag{9}$$

$$\dot{\vec{f}} = -g_3 e_2 \tag{10}$$

$$\widetilde{f}_{v} = -g_{4}e_{2} \tag{11}$$

$$\dot{\widetilde{A}}_{r1} = -g_5 e_2 \tag{12}$$

$$\dot{\tilde{A}}_{r2} = -g_6 e_2 \tag{13}$$

Here exists  $\dot{V}_A \leq -e^T \text{ Re } \leq 0$ . All signals are limited within finite boundaries. The parameters estimate of  $\hat{R}$ ,  $\hat{f}_c$ ,  $\hat{f}_v$ ,  $\hat{f}$ ,  $\hat{A}_{r1}$ ,  $\hat{A}_{r2}$  are also guaranteed within finite bo undaries. So *e*, *e* is converged to zero. That means the sta tes of system will converge to  $x_*$ asymptotically.

By the means of injecting the damp into PBC, the characteristics of transient process are improved. Rearranging the nonlinear feedback control law as:

$$u_{d} = -\frac{k_{1}}{L_{d}} x_{2} x_{d3} - k_{d} e_{1}$$

$$u_{q} = \dot{x}_{d2} + \frac{R}{L_{q}} x_{d2} + k_{1} (x_{1} + \lambda_{af}) x_{d3} - K_{q} e_{2}$$
(14)

By the control law (14), we get

$$\dot{V} = -e^{T}De - K_{d}e_{1}^{2} - K_{q}e_{2}^{2} \le 0$$
(15)

The system is still stable asymptotically. The fast convergent rate is feasible.

### **5. SIMULATION**

Parameters of PMLM studied in this paper are listed in table 1. Simulation parameters as Table 2

$R(\Omega)$	8.6
$L_q$ (mH)	6.0
$\lambda_{af}$ (V.s)	0.35
au (m)	0.031
Р	1
$M(\mathrm{Kg})$	1.635
$f_v$ (N.s/m)	0.1
$\mu$ (N/kg)	1
	$R ( \Omega )$ $L_{q} (mH)$ $\lambda_{af} (V.s)$ $\tau (m)$ $P$ $M (Kg)$ $f_{v} (N.s/m)$ $\mu (N/kg)$

Table 1 Linear motor parameter

Table 2 Simulation parameter

$f_s = 20N$
$f_c = 10N$
$A_r = 8.5N$
$\varphi = 0.05 \pi$
$v_{s} = 0.1m./s$
$f_l = 50N$

The speed tracking of PMLM to the step input are shown in Fig.2- Fig.7. The speed can track the step reference with ideal performance. The state variables  $i_d$  and  $i_q$  can also converge asymptotically to the reference state vector  $x_d$ . These results demonstrate the effect of suppressing of PBC to friction force and ripple force. Fig.8 and Fig.9 illustrate the tracking process with load variation. At the time of 1 second, let the load changes from 50N to 100N. The unity control scheme of PBC and the adaptive method guarantee the robustness to the load and the exact estimate of load. Fig.10 and Fig.11 illustrate the tracking process to the stair response and sinusoidal response.







#### Fig.11 Sinusoidal response curves of velocity

# 6. CONCLUSION

The based-passivity control strategy guarantees the stability globally and the convergence of the state errors asymptotically. It avoids the cancellation of system nonlinear and enhances the robustness. By keeping up the nonlinear terms that induce the system dynamics to equilibrium state and inject into damp, we design PBC to suppress the influence arising from load, friction and ripple force and give the system ideal performance. The application of adaptive technology realizes the robustness to parameters variation. Simulation demonstrates the effectiveness of PBC to the regulation of PMLM.

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