H_∞ State Estimation of Permanent Magnet Linear Synchronous Motors Using a Linear Matrix Inequality Approach

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Abstract—**This paper presents a novel state estimation method on permanent magnet linear synchronous motors (PMLSMs) through the H_∞ filtering viewpoint. The dynamic characteristic of the PMLSM is analyzed and the discrete state equation model for the PMLSM is developed. To give the velocity and position estimation of the linear motor, an H_∞ filtering problem which allows certain presence of uncertainties is constructed. The uncertainties include parameter variations, noises and unmodeled dynamics. Afterwards the H_∞ filtering problem is solved using a linear matrix inequality (LMI) approach. The simulation results show that the proposed method can predict the motor velocity and position with minimum error.**

Keywords—**H_∞ filtering, linear matrix inequality, permanent magnet linear synchronous motors, state estimation**

I. Introduction

With the emergence of high-performance power electronics and powerful digital signal processors (DSPs), permanent magnet linear synchronous motors (PMLSMs) are increasingly utilized in industrial automation, transportation, and domestic appliances [1-3]. PMLSMs have many advantages, such as quick response, high sensitivity, good tracking performance, etc., so they are gradually taking the place of combinations of rotary motors and lead-screws. To achieve high transient and steady performance, linear position sensors are required for the servo loop feedback. However, linear position sensors account for a large proportion of the total system cost. Furthermore, most linear position sensors have problems of difficult installation, low reliability, and are sensitive to alien surroundings, such as vibration and moisture. In some applications, inadequate space to do not permit the mounting of a linear position sensor.

Due to the above-mentioned disadvantages, the idea of eliminating mechanical sensors has attracted many research activities during the past decade. Many practical applications have been published. Unfortunately, most of them are based on speed control of rotary motors only. There is still little in recent literature that concerns with sensorless control of PMLSMs. Kalman filtering technique has once been used to estimate the velocity and position of a PMLSM, however, it requires the exact mathematical model of the plant.

In this paper, the authors proposed a novel state estimation method, which is based on H_∞ filtering. An H_∞ state estimation problem is regarded as a special case of H_∞ filtering problems. In the H_∞ filtering, the presence of some uncertainties is allowed, and robust performance can still be achieved. This paper first describes the dynamic model of the PMLSM and develops the discrete state equations. Then an H_∞ filtering problem is constructed to give state estimation expressions. To solve the H_∞ filtering problem, a linear matrix inequality approach is proposed. Finally, numerical simulation is made and the validity of the novel method is evaluated.

II. Mathematical Model

The permanent magnet linear motor used in this research is a sinusoidally commuted motor. According to the unification theory on motors, the dynamic characteristic of a PM linear motor can be described with Park's equations in *d-q* coordinate system, which moves synchronously with the mover, by applying Park's coordinates transformation [4].

$$v_q = Ri_q + p\lambda_q + \omega_s\lambda_d, \tag{1}$$
$$v_d = Ri_d + p\lambda_d - \omega_s\lambda_q, \tag{2}$$
$$\lambda_q = L_q i_q, \tag{3}$$
$$\lambda_d = L_d i_d + \lambda_{af}, \tag{4}$$

where v_d and v_q are stator voltages at direct-axis and quadrature-axis; i_d and i_q are stator currents at direct-axis and quadrature-axis; R is the stator resistance; p is the differential operator, d/dt; λ_d and λ_q are stator linkages at direct-axis and quadrature-axis; ω_s is the equivalent synchronous rotary angular velocity; L_d and L_q are stator inductances at direct-axis and quadrature-axis; λ_{af} is the permanent magnet linkage.

The following equation on the motor's total input power in *d-q* and *a-b-c* coordinate systems holds.

$$power = v_a i_a + v_b i_b + v_c i_c = 3(v_d i_d + v_q i_q)/2. \tag{5}$$

And the electromagnetic thrust and mechanical equations are described in (6) and (7), respectively.

$$F_e = 3N_p(\pi/\tau)(\lambda_d i_q - \lambda_q i_d)/2$$
$$= 3N_p(\pi/\tau)[\lambda_{af} i_q + (L_d - L_q)i_d i_q]/2, \tag{6}$$
$$F_e = F_L + B_v v + Mpv. \tag{7}$$

where F_e is the electromagnetical thrust; N_p is the number of pole pair(s); τ is the pole pitch; F_L is the load friction; v

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is the actual linear velocity; B_v is the damping coefficient associated with velocity; M is the mass of the moving part.

The constant electromagnetic energy principle is employed when the electromagnetic thrust expression is deduced, and linear motion is regarded as equivalent rotary motion, where ω_r is the equivalent mechanical angular velocity. Then these relationships hold: $\omega_r = v\pi/\tau$ and $\omega_s = N_p\omega_r$.

The above dynamic equations can be adapted to state-space format by choosing $X = [i_d, i_q, v, x]^T$ (x is the displacement) as state variables vector, $U = [v_d, v_q]^T$ as input vector, and $Y = [i_d, i_q]^T$ as output vector.

$$\dot{X} = f(X) + BU, \tag{8}$$

$$Y = h(X). \tag{9}$$

In order to implement numerical simulation and digital control, the continuous-time state equations must be changed into linear discrete-time state equations.

$$X(\mathrm{k}+1) = \Phi(\mathrm{k})X(\mathrm{k}) + G(\mathrm{k})U(\mathrm{k}), \tag{10a}$$

$$Y(\mathrm{k}) = H(\mathrm{k})X(\mathrm{k}). \tag{10b}$$

where k, k+1 denote the k -th and the k+1 -th sampling moment, respectively. $\Phi(\mathrm{k})$ is the state transition matrix. The relationships between matrices $\Phi(\mathrm{k})$, $G(\mathrm{k})$, $H(\mathrm{k})$ and the matrices in continuous-time system are as follows.

$$\Phi(\mathrm{k}) = e^{F(kT_s)\cdot T_s} \approx I + F(\mathrm{k}T_s)\cdot T_s, \tag{11}$$

$$G(\mathrm{k}) = \int_0^{T_s} e^{F(kT_s)\cdot t} B(kT_s)dt \approx B(\mathrm{k}T_s)\cdot T_s, \tag{12}$$

$$H(\mathrm{k}) = H(\mathrm{k}T_s); \tag{13}$$

where I is a 4×4 unit matrix; $F(t)$ is the Jacobian matrix of $f(X)$,.

III. THE H∞ FILTERING

The H∞ filtering technique has been widely studied for the benefit of different time and frequency domain properties. An H∞ filter is designed such that the H∞ norm the system, which reflects the worst-case "gain" of the system, is minimized [5]. J. S. Kim, I. W. Yang, Y. S. Kim, and Y. J. Kim [6] use the H∞ filter as an observer of the state feedback controller. Experimental results show that the H∞ filter has excellent estimating property. It has been used in shaft vibration suppression control of the industrial motor drive system. K. Reif, F. Sonnemann, and R. Unbehauen [7] also propose an H∞ filtering observer for continuous-time nonlinear systems. The remaining nonlinear terms are treated as uncertainties and are implemented in the proposed observer design by a worst case valuation in such a way that these uncertainties can be tolerated. To examine the practical usefulness of the proposed observer they applied it to an induction motor for the estimation of the rotor flux and the angular velocity. U. Shaked and Y. Theodor [8] give a tutorial to H∞-optimal estimation of linear, time-varying processes, in both the continuous- and the discrete-time cases. Four illustrative examples are given that demonstrate the various estimation schemes.

In this paper, the state estimation of the motor is regarded as a special case of H∞ filtering problems. Considering the system noises and the measurement noises, the discrete-time equations are rewritten in a general form of H∞ control.

$$X(\mathrm{k}+1) = \Phi(\mathrm{k})X(\mathrm{k}) + B_1(\mathrm{k})W(\mathrm{k}) + G(\mathrm{k})U(\mathrm{k}), \tag{14a}$$

$$Z(\mathrm{k}) = C_1X(\mathrm{k}), \tag{14b}$$

$$Y(\mathrm{k}) = H(\mathrm{k})X(\mathrm{k}) + D_{21}(\mathrm{k})W(\mathrm{k}). \tag{14c}$$

where $C_1 = I_{4\times4}$, i.e., $Z(\mathrm{k}) = X(\mathrm{k})$.

Assume the H∞ filter has the following structure,

$$X_f(\mathrm{k}+1) = \Phi(\mathrm{k})X_f(\mathrm{k}) + G(\mathrm{k})U(\mathrm{k}) + K(\mathrm{k})[Y(\mathrm{k}) - H(\mathrm{k})X_f(\mathrm{k})], \tag{15a}$$

$$Z_f(\mathrm{k}) = C_1X_f(\mathrm{k}), \tag{15b}$$

and it has the same initial states with the practical system, i.e. all are zero.

Subtracting (15) from (14), we get the errors dynamic model, which can be written in the form of H∞ controller.

$$X_e(\mathrm{k}+1) = \Phi(\mathrm{k})X_e(\mathrm{k}) + B_1(\mathrm{k})W(\mathrm{k}) + U_e(\mathrm{k}), \tag{16a}$$

$$E(\mathrm{k}) = C_1X_e(\mathrm{k}), \tag{16b}$$

$$Y_e(\mathrm{k}) = H(\mathrm{k})X_e(\mathrm{k}) + D_{21}(\mathrm{k})W(\mathrm{k}), \tag{16c}$$

$$U_e(\mathrm{k}) = -K(\mathrm{k})Y_e(\mathrm{k}). \tag{16d}$$

where $X_e(\mathrm{k}) = X(\mathrm{k}) - X_f(\mathrm{k})$, $E(\mathrm{k}) = Z(\mathrm{k}) - Z_f(\mathrm{k})$.

Now we can solve the controller $K(\mathrm{k})$ in the error system using a linear matrix inequality approach.

IV. LINEAR MATRIX INEQUALITY APPROACH

There are three approaches to H∞ filtering [5]:

1) algebraic Riccati equation (ARE) approach;
2) frequency domain approach;
3) linear matrix inequality (LMI) approach.

However, the first two methods require very strict constraints, or have computational complexity. On the other hand, the linear matrix inequality approach has proved an effective method to solve convex optimization problem using numerical interior point technique. Instead of arriving at an analytical solution, the LMI approach is to reformulate a given problem to verifying whether an LMI is solvable or to optimizing functionals over LMI constraints [9].

For discrete-time system, the solution to an H∞ controller has turned to solving the following three LMI's, and the final solutions $R = R^T$ and $S = S^T$ make the H∞ performance index γ reach its minimum or optimum value [10].

$$\begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} ARA^T - R & ARC_1^T & B_1 \\ C_1RA^T & -\gamma I + C_1RC_1^T & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} \begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A^TSA - S & SB_1A^T & C_1^T \\ B_1^TSA & -\gamma I + B_1^TSB_1 & D_{11}^T \\ C_1 & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0. \tag{19}$$

where N_R and N_S denote bases of the null spaces of (B_2^T, D_{12}^T) and (C_2, D_{21}), respectively.

The LMI approach is divided into 6 steps in numerical simulation [11].

S1. Solve the LMIs in (17)~(19) and get a feasible solution (R, S, γ);

S2. Decompose $I - RS$ to two matrices M and N with full column rank using singular value decomposition technique, $MN^{\mathrm{T}} = I - RS$;

S3. Let $D_K = (D_{12}^{+} D_{12}) D_0 (D_{21} D_{21}^{+})$, where D_0 is an arbitrary matrix that meets $\sigma_{\max}(D_{11} + D_{12} D_0 D_{21}) < \gamma$, the superscript "+" denotes the Moore-Penrose pseudo-inverse of a matrix;

S4. Solve the following equations and get their solutions B_K and C_K:

$$NB_K = -SB_2 D_K + K_B^T , \tag{20}$$

$$C_K M^T = -D_K C_2 R + K_C . \tag{21}$$

S5. Calculate A_K(see, e.g., [11]);

S6. Obtain the "central" controller $K(k) = D_K + C_K (sI - A_K)^{-1} B_K$.

This approach can be easily programmed using MATLAB's LMI control toolbox [10]. It is more efficient than DGKF's "2-Riccati equations" method.

V. SIMULATION RESULTS

The parameters of the PMLSM are listed in the appendix. The discrete-time state equations of the motor in *d-q* coordinate system are used for numerical simulation. The unit velocity step response is investigated with a step of T_s = 5μs. The simulation results are shown in Fig. 1 through Fig. 6.

The instantaneous input voltages and motor currents are "measured" in *d-q* coordinate system. Their waveforms are shown in Fig. 1 and Fig. 2, respectively. In Fig. 3 the estimated velocity curve is plotted in comparison with the actual value. In Fig. 4 the estimated position and actual position are also compared. From Fig. 5 we know the error between estimated and actual velocity is very small. The maximum relative error is about 1.2%. The error between estimated and actual position is also drawn in Fig. 6. The position estimation error is less than 6 μm.

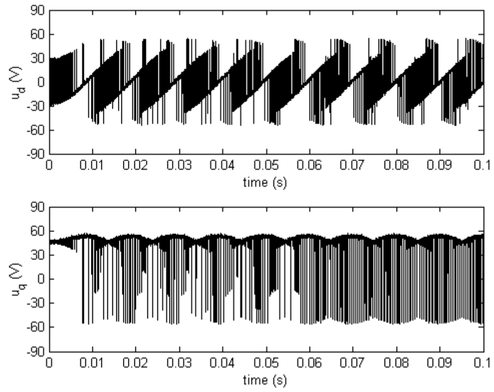

Fig. 1. Measured input voltages
Upper: *d*-axis voltage; Lower: *q*-axis voltage

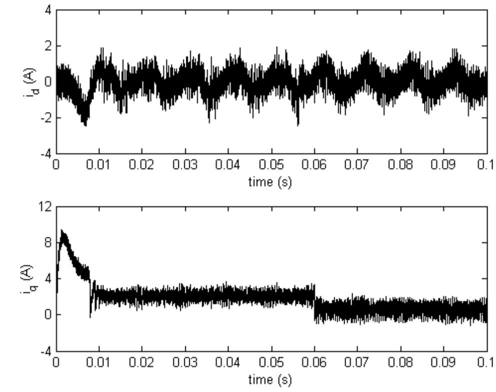

Fig. 2. Measured motor currents
Upper: *d*-axis current; Lower: *q*-axis current

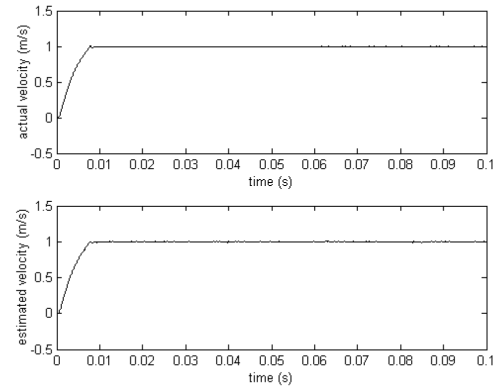

Fig. 3. Velocity step response curves
Upper: actual value; Lower: estimated value

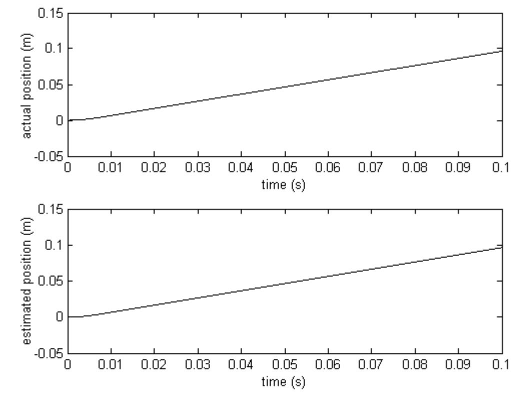

Fig. 4. Position curves under closed-loop velocity step-response trial
Upper: actual position; Lower: estimated position

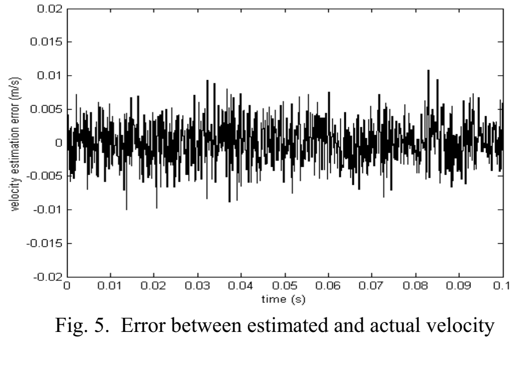

Fig. 5. Error between estimated and actual velocity

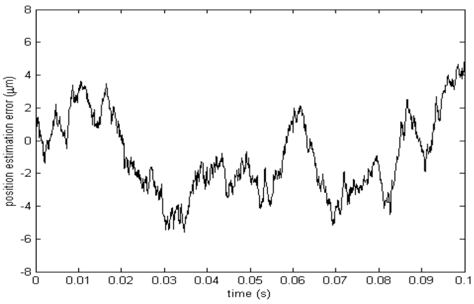

Fig. 6. Error between estimated and actual position

VI. CONCLUSION

Results show that the state estimation can be considered in an H_∞ filtering sense. This method can give accurate velocity and position information in spite of uncertainties in the system. So the estimation algorithm is robust to parameter variations and external disturbances. The LMI approach is effective to solve H_∞ filtering problem, which avoids treating algebraic Riccati equations. Simulation results show that the estimation errors are very small and the need for velocity and position sensors can be eliminated.

APPENDIX

The motor parameters are listed below:
phase resistance R: 8.6 ohm;
phase synchronous inductance L: 6 mH;
permanent magnet flux linkage λ: 0.35 V·s;
pole pitch of the permanent magnets τ: 0.031 m;
pole pair number N_p: 1;
total mass of moving parts M: 1.635 kg;
viscous damping coefficient B_v: 0.1 N·s/m;
load F_L: 10 N

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