H_∞ Robust Control of Permanent Magnet Linear Synchronous Motor in High-Performance Motion System with Large Parametric Uncertainty

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***Abstract*- In order to meet the demands of high-acceleration/deceleration and high-precision motion profiles, many motion control systems began to use direct-drive linear motor as the prime motion actuator. This arrangement has the advantage of providing high-performance motions with reduced mechanical components, but its major drawback is the effect of load variation on the overall system control. Unlike conventional ball-screw drive, a linear direct-drive system eliminates the mechanical couplings, rotary-to-linear translators, and reduction gears. Under this arrangement, any change or disturbance in the load will be directly reflected back to the motor and the control system. This will cause large deterioration in the motion profile. In this paper, the authors proposed to use an H_∞ robust-controller to overcome the load uncertainty problem. In the investigation, a Permanent Magnet Linear Synchronous Motor (PMLSM) with large parametric uncertainty is chosen as the target study. First, the state space equations of the motor are established. Then the H_∞ control theory is applied to design a robust controller which allows mass variation of the moving part ranging from 0 to 100 percent of nominal load. To minimize the error between the actual response and the reference, the controller parameters are optimized using genetic algorithms (GA). The simulation and experimental results both show that the system can achieve robust performance under large load variations. Thus, the proposed method is an effective mean of combating load variations and load disturbances in high-performance direct-drive systems.**

I. INTRODUCTION

With the recent advances of computer technologies, control theories and material technologies, more and more linear motors are employed in applications ranging from mass transportation to factory automation [1, 2, 3]. Among these applications, PMLSMs are frequently used. This is due to their simple structure, ease of manufacture and simple control. In this paper, we study the case of PMLSM used as a transportation system in factory automation. For this purpose, the mass of the moving part varies frequently under no-load or full-load conditions. The friction of the motion system also varies frequently and significantly. Therefore it is necessary to design a robust controller that is insensitive to the variations of mass and friction, to ensure that the system is stable system under arbitrary loads.

Obviously, it is difficult to meet above specifications using conventional PID controllers. PID controllers cannot make the system stable under certain conditions. If applying linear system theory can solve the control problem, then the accurate mathematical model of the system is required. However, the variation of mass and friction causes parametric uncertainty. In this paper, an H_∞ control strategy is proposed to overcome the above-mentioned disadvantages. In H_∞ control system, the exact model is not required, and some uncertainty is allowed [4, 5, 6].

This paper analyses the system dynamic characteristics, deduces the state space model of the system, and designs an optimal or sub-optimal H_∞ controller for the PMLSM from the H_∞ control theory. To minimize the error between the actual velocity response and the ideal velocity reference, the controller parameters is finally optimized using a kind of genetic algorithm. The simulation and experimental results show that the proposed control method is effective and highly robust, and it is very suitable for implementation in direct-drive linear motion systems with large load variations.

II. Dynamic Model of the System

Firstly, the *d-q* dynamic model for the PMLSM is studied. The *d-q* coordinate system is a "rotating" reference frame that moves at synchronous speed. The flux linkage equations are as follow.

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_f \end{bmatrix} = \begin{bmatrix} L_d & 0 & M_f \\ 0 & L_q & 0 \\ 0 & 0 & M_f \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_f \end{bmatrix} \tag{1}$$

where Ψ_d, Ψ_q, Ψ_f are the flux linkage of direct axis, quadrature axis and permanent magnet, respectively. L_d, L_q, M_f and i_d, i_q, i_f are inductance and current of *d*-, *q*-axis and equivalent permanent magnet, respectively. For surface PMs, we have $L_d = L_q$ [2]. Ψ_f, M_f and i_f are all constant.

The voltage equations of the PMLSM are shown in (2):

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = \begin{bmatrix} R + pL_d & -\omega L_q \\ \omega L_d & R + pL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega M_f i_f \end{bmatrix} \tag{2}$$

where u_d, u_q are *d*-, *q*-axis armature voltage, R is phase resistance, ω is equivalent "electrical angular velocity", $\omega = v\pi/\tau$, v is the velocity of moving part, τ is the pole pitch, p is derivative operator, $p = d/dt$.

The electromagnetic thrust is

$$F_m = P_{elm}/v = 1.5p_n(\pi/\tau)(\Psi_d i_q - \Psi_q i_d)$$

$$= 1.5p_n(\pi/\tau)[\Psi_f i_q + (L_d - L_q) i_d i_q] \tag{3}$$

where F_m is the output electromagnet thrust, P_{elm} is the electromagnet power, p_n is the number of pole-pair(s). In this paper, $p_n = 1$.

Lastly, the motion equation of the system is obtained according to Newton's mechanics law:

$$dv/dt = (F_m - f - R_v v)/m \tag{4}$$

where m is the mass of moving part including the load, f is the total friction, R_v is the damper coefficient associated with velocity.

When $i_d = 0$ control scheme is applied, the *d*-axis flux linkage is equal to the permanent magnet flux linkage. The dynamic model of the system can be simplified. Considering the velocity v, *q*-axis current i_q and total friction coefficient μ as state variables, we obtain the state equation and output equation of the system as follow.

$$\dot{x} = Ax + Bu \tag{5}$$

$$y = Cx \tag{6}$$

where

$$x = \begin{bmatrix} v \\ i_q \\ \mu \end{bmatrix},$$

$$A = \begin{bmatrix} -\dfrac{R_v}{m} & 1.5\dfrac{\pi}{\tau}\dfrac{\Psi_f}{m} & -1 \\ -\dfrac{\pi}{\tau}\dfrac{\Psi_f}{L_q} & -\dfrac{R}{L_q} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \dfrac{1}{L_q} \\ 0 \end{bmatrix},$$

$u = u_q$,

$C = [1 \ \ 0 \ \ 0]$,

where the mass of the moving part when under rating load is up to 10 times of that when under no load.

III. The Proposed Method

In H_∞ theory, J. C. Doyle *et al.* have proved that a standard H_∞ problem can be solved by finding the unique stabilizing solutions to two algebraic Riccati equations (AREs) [7]. Genetic algorithms have been found an effective method for searching the global optimal solutions without the derivative information [8]. It can avoid going to a local optimal solution by random searching strategy. In this section, we will introduce how to design an H_∞ controller that is robust to parameter uncertainty. Then the controller parameter is optimized using genetic algorithm.

A. H_∞ Controller Design

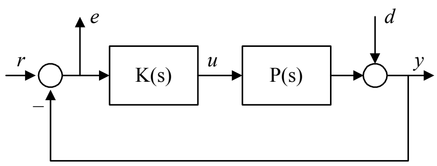

Fig. 1. The control system block diagram

Fig.1 shows the control system block diagram. We define $S(s)$ as the sensitivity function.

$$S(s) = [I + K(s)P(s)]^{-1} \tag{7}$$

$S(s)$ is the closed-loop transfer function from disturbance d to error e. So the influence of disturbance to control error can be reduced when tuning down the gain of $S(s)$. We introduce a weight function $W(s)$ to error output and redraw the system block diagram in the form of standard H_∞ problem, which is shown in Fig.2.

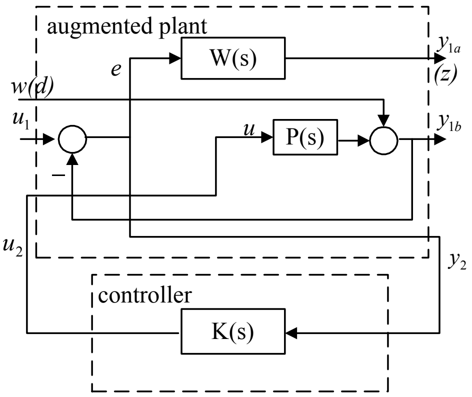

Fig. 2. The standard H_∞ problem

From disturbance w, i.e. d, to weighted error output z, i.e. y_{1a}, the closed-loop transfer function $T_{zw}(s)$ is

$$T_{zw}(s)= W(s)S(s) \tag{8}$$

The optimal H_∞ control problem is to find a feedback controller $K(s)$, which can make the system internally stable and minimize the H_∞ norm of $T_{zw}(s)$, i.e.,

$$\min_{K stabilizing} \left\| T_{zw}(s) \right\|_\infty = \gamma_0 \tag{9}$$

In practice, usually we can only get a sub-optimal solution that makes $\|T_{zw}(s)\|_\infty = \gamma > \gamma_0$. Considering $W(s)$ as part of the system, the model of the augmented controlled plant has the following form.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \end{aligned} \tag{10}$$

Note

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right].$$

By using the DGKF method [7], when the augmented plant $G(s)$ meets certain constraints, we can solve the following algebraic Reccati equations:

$$A^T X + XA + X(B_1 B_1^T - B_2 B_2^T)X + C_1^T C_1 = 0 \tag{11}$$

$$AY + YA^T + Y(C_1^T C_1 - C_2^T C_2)Y + B_1 B_1^T = 0 \tag{12}$$

If (11) has positive semi-definite solution $X \geq 0$ that makes $A+(B_1B_1^T - B_2B_2^T)X$ stable, (12) has positive semi-definite solution $Y \geq 0$ that makes $A+(C_1^TC_1 - C_2^TC_2)Y$ stable, and $\lambda_{max}(XY)<1$, then the (sub-)optimal H_∞ controller $K(s)$ is obtained.

$$K(s) = \begin{bmatrix} A + B_1 B_1^T X - (1-XY)^{-1} Y C_2^T C_2 - B_2 B_2^T X & -(1-XY)^{-1} Y C_2^T \\ B_2^T X & O \end{bmatrix} \tag{13}$$

In practice, we can handle the above procedures with the help of numeric calculation and simulation tools, e.g. MATLAB.

B. Genetic Algorithm Optimization

Once the sub-optimal H_∞ controller is obtained. We can use genetic algorithm to search a best parameter configuration for the controller by the following steps.

Step 1. Give a reasonable range of these parameters, then a population with size N is initialized randomly.

Step 2. Check if the population meets the fitness function. Here we define the fitness function f_{fit} as

$$f_{fit} = \min \int_0^\infty e^2 dt \tag{14}$$

where e is the error between the actual output and the reference.

Step 3. Selection and reproduction. The individuals with better fitness will be selected into next generation.

Step 4. Crossover with probability P_c.

Step 5. Mutation with probability P_m.

After repeating step 2 to step 5 for a few iterations (generations), the best controller will be obtained.

IV. Simulation and Hardware Implementation

The proposed scheme was verified by both numerical simulation and experiment with a PC computer and a DSP control card from dSPACE company.

A. Simulation Results

In this paper, the weight function is selected as $W(s) = 0.5(s+1)/(0.5s+1)$. The motor parameters are listed in the appendix. Assuming the mass as 5 times of moving part and substituting these parameters into the system equations, we can now design the H$_\infty$ controller using robust control toolbox of MATLAB [9]. The ITAE type I is considered as standard model with the third order and natural frequency 10 rad/sec. After 8 γ iterations, the optimal H$_\infty$ controller is solved. Then we can use GA to optimize the controller parameters. Here the population size N is chosen as 25. The crossover probability P_c and mutation probability P_m are 0.9 and 0.005, respectively. After 20 generations, the best configuration of the H$_\infty$ controller parameters is reached.

$\gamma = 0.9922$,

$$K(s) = 0.79\frac{(s+1.716\times10^4 \pm 2.669\times10^4 i)}{(s+2.937\times10^4 \pm 2.935\times10^4 i)}\times \frac{(s+1447)(s+53.09)}{(s+8.75\pm11.77i)(s+4.651)}.$$

Fig.3 shows the simulation result of velocity step response with mass variation range from 0 to 100 percent of nominal load. The system has excellent performance when the mass is 5 times of system mass, the overshoot is very little and the settling time is short. When the mass decreases, the overshoot goes down but the settling time increases. On the contrary, when the mass increases, the overshoot increases, and the settling time also increases. However, the closed-loop system is stable no matter how it is under any load. So the controller is robust even if the controlled plant has large parametric uncertainty.

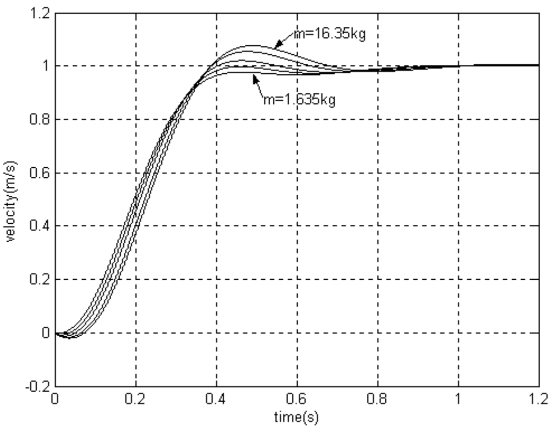

Fig. 3. Simulated velocity step responses with different masses

B. Hardware Implementation

The overall experimental system setup is shown in Fig.4. An incremental linear encoder is used to measure the actual position of the moving part as a reference. An integrated IGBT circuit CPV363M4K serves as the power drive module. An IC IR2132 is used to drive the gates of the IGBT [10]. The main circuit is isolated from the controller board by opto-couplers. Since it is a direct digital control method, the whole system is simple and clear. Fig.5 shows the schematic diagram of the proposed drive system. The internal loop is controlled by a hysteresis current controller. It has a highly rigid response to current change. So the PMLSM can almost operate in sinusoidal current mode.

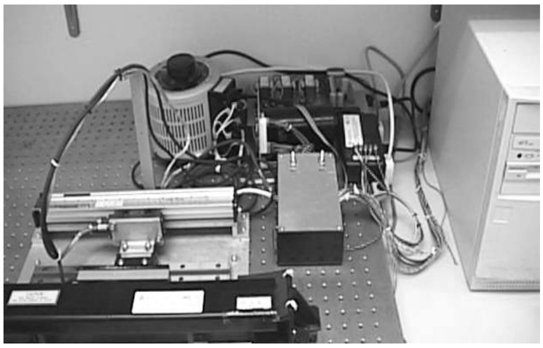

Fig. 4. The overall setup of experimental system

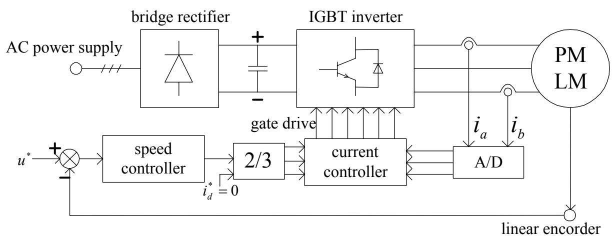

Fig. 5. The schematic diagram of the drive system

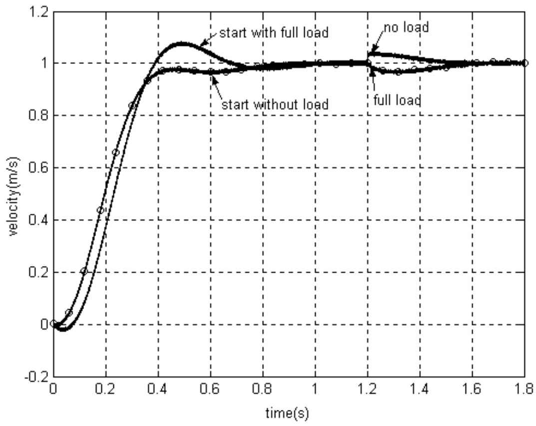

Fig. 6. Actual velocity step responses under no-load and full load

–o– starting without load, at 1.2s full load is suddenly added

—— starting with full load, at 1.2s load is suddenly emptied

In Fig.6, the actual velocity step responses under no-load and full load are given. Two worst cases have been examined. One is that the motor starts without load, then full load is suddenly exerted at 1.2s. The other is that the motor starts with full load, then the load is suddenly emptied at 1.2s. Both are coincident with the simulation results. The system is stable no matter how much load it carries. However, if this plant is controlled by a PID controller, the PID coefficients must be modified frequently with the variation of the total mass in real time. And, the whole system will be unstable when it is under full load. Even if it is stable, the step response is worse with large overshoot and long settling time.

V. Conclusions

This paper describes the use of H_∞ controller in PMLSM drive in the application of automated transport system with large load variations and friction disturbances. A kind of GA optimization technique is also used to find the best controller parameter configuration. The simulation and experimental results show that the optimized H_∞ controller has a robust performance. It can make the closed-loop system internally stable and can handle large parametric uncertainty. The proposed controller is superior to PID controller and other similar linear controllers. Both the simulated result and the implementation result show that H_∞ design method and GA optimization technique are very suitable for PMLSM drives with model or parametric uncertainty or disturbance spectrum uncertainty.

VI. Appendix

The motor parameters used in the simulation and the actual implementation are listed as follows:

phase resistance R: 4.5 ohm;

phase synchronous inductance L: 3.0 mH;

permanent magnet flux linkage Ψ_f: 0.3438 $V\cdot s\cdot rad^{-1}$;

pole pitch of the permanent magnets τ: 0.03048 m;

number of pole-pair(s) p_n: 1;

total mass of moving part excluding load m: 1.635 kg;

viscous damping coefficient R_v: 0.1 $N\cdot s\cdot m^{-1}$;

total friction coefficient μ: 1 $N\cdot kg^{-1}$.

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