H_∞ Control of Permanent Magnet Linear Motor in Transportation Systems

Norbert C. Cheung[1] Yuan-Rui Chen[2] Jie Wu[2]
[1] Department of Electrical Engineering, Hong Kong Polytechnic University,
Hung Hom, Kowloon, Hong Kong SAR, China
E-mail: eencheun@polyu.edu.hk
[2] Electric Power College, South China University of Technology
Guangzhou 510640, China
E-mail: chenyr@21cn.com

***Abstract*-A robust H_∞ controller for Permanent Magnet Linear Motor (PMLM) in transportation systems is designed in this paper. First, the state space equations of the motor are established. Then the H_∞ control theory is applied to design a robust controller which allows mass variation of the moving part ranging from 0 to 100 percent of nominal load. The simulation and experimental results show that the system can achieve robust performance under either no load or nominal load.**

I. INTRODUCTION

With the newly development of computer, control theory and material technology, more and more linear motors are cast into applications in the areas of transportation and factory automation. Among these applications, PMLMs are frequently used due to their simple structure, ease to manufacture and control. In this paper, we study the case of PMLM used in transportation system in factory automation. The mass of the moving part varies frequently due to the system is often under load ranging from 0 to 100 percent of nominal load, and the friction also varies consequently. So it is necessary to design a robust controller, which is not very sensitive to the variation of mass and friction, to ensure the system stable under arbitrary load within rating range.

Obviously, it is difficult to meet above specifications using conventional PID controller. Sometimes it could not make the system stable. If the linear system theory is applied, then accurate mathematical model of the system is required. However, the variation of mass and friction causes parametric uncertainty. Fortunately, H_∞ control theory gets rid of these disadvantages. The exact model is no more required, and some uncertainty is allowed in the system. So H_∞ theory is suitable for control engineering practice.

This paper analyses the system dynamic characteristics, deduces the state space model of the system, and designs an optimal H∞ controller for the PMLM from the H∞ control theory. The simulation and experimental results are given to show the effectiveness of the controller and the robustness of the closed-loop system.

II. DYNAMIC MODEL OF THE SYSTEM

The *d-q* dynamic model is frequently used for sinusoidally excited PMLMs. The d-q coordinate system is a "rotating" reference frame that moves at synchronous speed. The flux linkage equations are as follow:

$$\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_f\end{bmatrix}=\begin{bmatrix}L_d & 0 & M_f\\ 0 & L_q & 0\\ 0 & 0 & M_f\end{bmatrix}\begin{bmatrix}i_d\\ i_q\\ i_f\end{bmatrix} \quad (1)$$

where Ψ_d, Ψ_q, Ψ_f are the flux linkage of direct axis, quadrature axis and permanent magnet, respectively. L_d, L_q, M_f and i_d, i_q, i_f are inductance and current of *d*-, *q*-axis and equivalent permanent magnet, respectively. Ψ_f, M_f and i_f are all constant.

The voltage equations of the PMLM are listed below:

$$\begin{bmatrix}u_d\\ u_q\end{bmatrix}=\begin{bmatrix}R+pL_d & -\omega L_q\\ \omega L_d & R+pL_q\end{bmatrix}\begin{bmatrix}i_d\\ i_q\end{bmatrix}+\begin{bmatrix}0\\ \omega M_f i_f\end{bmatrix} \quad (2)$$

where u_d, u_q are *d*-, *q*-axis armature voltage, R is phase resistance, ω is equivalent "electrical velocity", $\omega = v\pi/\tau$, v is the velocity of moving part, τ is the pole pitch, p is derivative operator, $p = d/dt$.

The electromagnetic thrust is

$$F_m = P_m/v = 1.5p_0(\pi/\tau)(\Psi_d i_q - \Psi_q i_d)$$
$$= 1.5p_0(\pi/\tau)[\Psi_f i_q + (L_d - L_q)i_d i_q] \quad (3)$$

where F_m is the output electromagnet thrust, P_m is the electromagnet power, p_0 is the number of pole-pair(s). In this paper, $p_0 = 1$.

At last, the motion equation of the system is obtained according to Newton's mechanics law:

$$dv/dt = (F_m - f - R_v v)/m \quad (4)$$

where m is the mass of moving part including the load, f is the total friction, R_v is the damper coefficient associated with velocity.

When $i_d = 0$ control scheme is applied, the *d*-axis flux linkage is equal to the permanent magnet flux linkage. The dynamic model of the system can be simplified. Considering the velocity v, *q*-axis current i_q and total friction coefficient μ as state variables, we obtain the state equation and output equation of the system as follow:

$$\dot{x} = Ax + Bu \quad (5)$$

$$y = Cx \quad (6)$$

where

$$x=\begin{bmatrix}v\\ i_q\\ \mu\end{bmatrix},$$

$$A=\begin{bmatrix} -\dfrac{R_v}{m} & 1.5\dfrac{\pi}{\tau}\dfrac{\psi_f}{m} & -1 \\ -\dfrac{\pi}{\tau}\dfrac{1}{L_q} & -\dfrac{R}{L_q} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} 0 \\ \dfrac{1}{L_q} \\ 0 \end{bmatrix},$$

$$u=u_q,$$

$$C=\begin{bmatrix}1 & 0 & 0\end{bmatrix},$$

where the mass of the moving part when under rating load is up to 10 times of that when under no load.

III. H_∞ CONTROL ALGORITHM

Fig.1 shows the control system block diagram. We define $S(s)$ as the sensibility function.

$$S(s) = [I+ K(s)P(s)]^{-1} \tag{7}$$

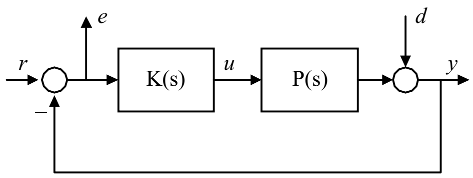

Fig.1 The control system block diagram

Actually $S(s)$ is the closed-loop transfer function from disturbance d to error e. So the influence of disturbance to control error can be reduced when tuning down the gain of $S(s)$. We introduce a weight function $W(s)$ to error output and redraw the system block diagram in the form of standard H_∞ problem, which is shown in Fig.2. From disturbance w to weighted error output z, the closed-loop transfer function $T_{zw}(s)$ is

$$T_{zw}(s)= W(s)S(s) \tag{8}$$

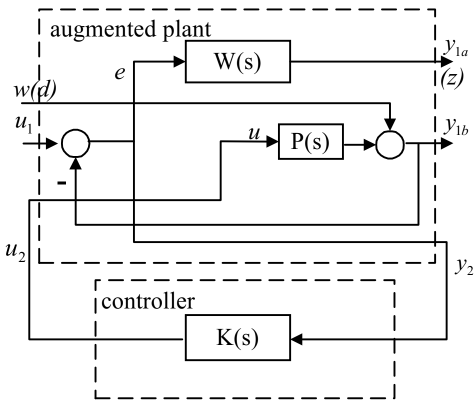

Fig.2 The standard H_∞ problem

The optimal H_∞ control problem is to find a feedback controller $K(s)$, which can make the system internally stable and minimize the H∞ norm of $T_{zw}(s)$, i.e.,

$$\min_{K stabilizing} \left\| T_{zw}(s) \right\|_\infty = \gamma_0 \tag{9}$$

Note

$$G=\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right].$$

By using the DGKF method, when the augmented plant $G(s)$ meets certain constrains, we can solve the following algebra Reccati equations:

$$A^T X + XA + X(B_1B_1^T - B_2B_2^T)X + C_1^T C_1 = 0 \tag{11}$$

$$AY + YA^T + Y(C_1^T C_1 - C_2^T C_2)Y + B_1B_1^T = 0 \tag{12}$$

If (11) has positive semi-definite solution $X\geq 0$ that makes $A+(B_1B_1^T-B_2B_2^T)X$ stable, (12) has positive semi-definite solution $Y\geq 0$ that makes $A+(C_1^TC_1-C_2^TC_2)Y$ stable, and the maximum eigenvalue of XY meets $\lambda_{max}(XY)<1$, then the optimal H_∞ controller $K(s)$ can be obtained.

$$K(s)=\left[\begin{array}{cc} A+B_1B_1^T X-(1-XY)^{-1}YC_2^T C_2 - B_2B_2^T X & -(1-XY)^{-1}YC_2^T \\ B_2^T X & O \end{array}\right] \tag{13}$$

In practice, we can handle the above procedures with the help of numeric calculation and simulation tools, e.g. MATLAB.

IV. SIMULATION AND EXPERIMENTAL RESULTS

The proposed scheme was verified by both numerical simulation and experiment with a PC computer and a DSP control card from dSPACE.

A Simulation Results

In this paper, the weight function is selected as $W(s) = 0.5(s+1)/(0.5s+1)$. Fig.3 shows its magnitude-frequency and phase-frequency characteristics. The motor parameters are listed in the appendix. Assuming the mass as 5 times of moving

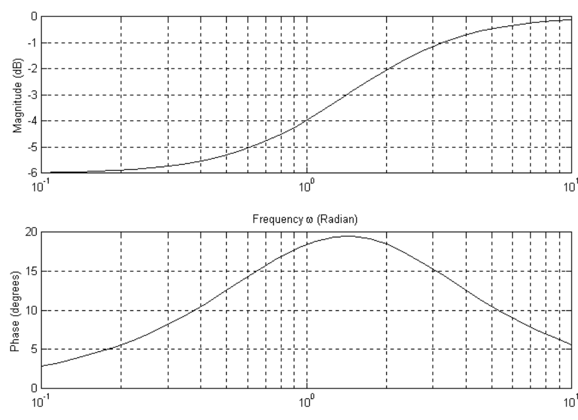

Fig.3 The Bode plot of weight function $W(s)$

part and substituting these parameters into the system equations, we can now design the H_∞ controller using robust control toolbox of MATLAB. The ITAE type I is considered as standard model with the third order and natural frequency 10 rad/sec. After 8 γ iterations, the optimal H_∞ controller is solved.

$\gamma = 0.9922$,

$$K(s) = 1.275\frac{(s+1.152\times10^4 \pm 1.987\times10^4 i)}{(s+2.064\times10^4 \pm 2.062\times10^4 i)}\times \frac{(s+1407)(s+26.64)}{(s+8.75\pm11.77i)(s+4.651)}$$

Fig.4 shows the simulation result of step response with mass variation range from 0 to 100 percent of nominal load. The system has excellent performance when the mass is 5 times of system mass, the overshoot is very little and the settling time is short. When the mass decreases, the overshoot goes down but the settling time increases. On the contrary, when the mass increases, the overshoot increases, and the settling time also increases. However, the closed-loop system is stable no matter how it is under any load. So the controller is robust even if the controlled plant has large parametric uncertainty.

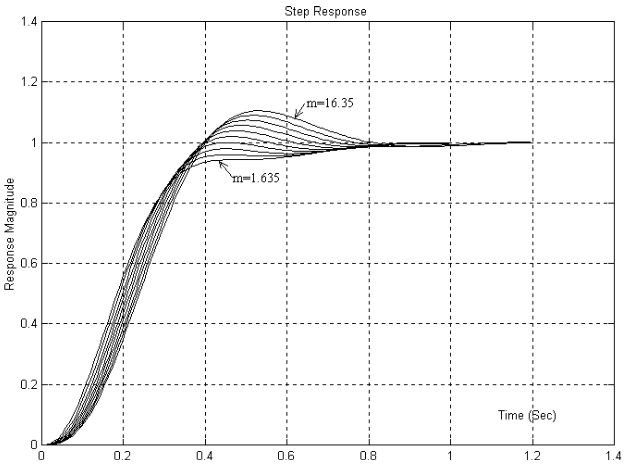

Fig.4 Step responses with different masses

B Hardware Implementation

The overall experimental system setup is shown in Fig.5. An incremental linear encoder is used to measure the actual position of the moving part as a reference. An integrated IGBT circuit CPV363M4K serves as the power drive module. An IC IR2132 is used to drive the gates of the IGBT. The main circuit is isolated from the controller board by opto-couplers. Since it is a direct digital control method, the whole system is simple and clear. Fig.6 shows the schematic diagram of the proposed drive system. The internal loop is controlled by a hysteresis current controller. It has a highly rigid response to current change. So the PMLM can almost operate in sinusoidal current mode.

In Fig.7, the actual position step responses under no-load and full load are given. They are coincident with the simulation results. The system is stable no matter how much load it carries.

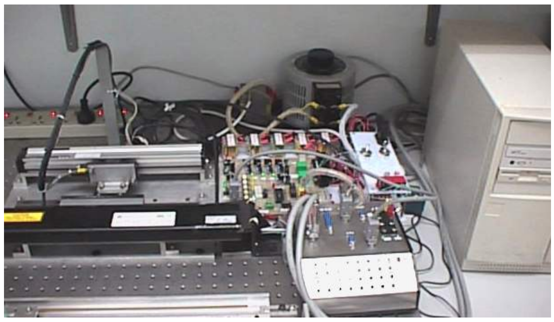

Fig.5 The overall setup of experimental system

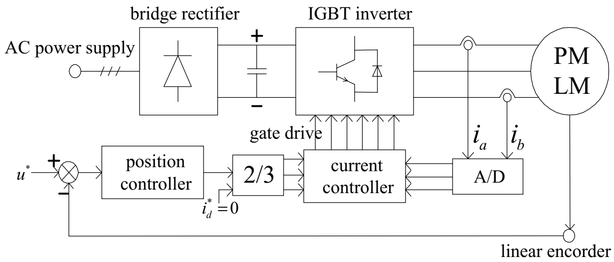

Fig.6 The schematic diagram of the drive system

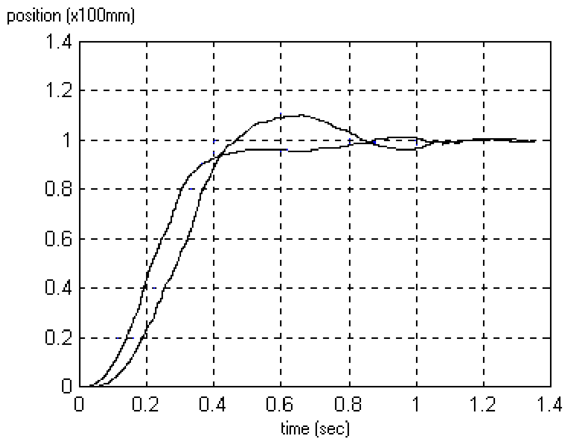

Fig.7 Actual step responses under no-load and full load

However, if this plant is controlled by a PID controller, the coefficients must be modified with the variation of the total mass. And, the whole system will be unstable when it is under full load. Even if it is stable, the step response is worse with large overshoot and long settling time.

V. Conclusions and Discussions

The simulation and experimental results show that the H_∞ controller has robust performance. It can make the closed-loop system stable and can handle large parametric uncertainty. The proposed controller is superior to PID controller and other similar linear controllers. By employing the loopshaping to the controlled plant in frequency domain, the H_∞ design method is very suitable for PMLM drives with model or parametric uncertainty or disturbance spectrum uncertainty. Most physical system can achieve ideal performance when controlled by a proper H_∞ controller.

Further investigation should be conducted that a load is suddenly thrown to the system while the motor is running. The dynamic performance of the whole system will be evaluated under more uncertainties. This will be very useful to practical transportation systems.

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APPENDIX

Below are the motor parameters used in the simulation and the actual implementation:
phase resistance R: 8.6 ohm;
phase synchronous inductance L: 6 mH;
permanent magnet flux linkage λ: 0.35 Vs;
pole pitch of the permanent magnets τ: 0.031 m;
number of pole-pair(s) p_0: 1;
total mass of moving part excluding load m: 1.635 kg;
viscous damping coefficient B_v: 0.1 Ns/m;
total friction coefficient μ: 1 N/kg;
maximum travelling distance: 0.18m.

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