MODELLING OF A NONLINEAR SOLENOID TOWARDS THE DEVELOPMENT OF A PROPORTIONAL ACTUATOR.

M. F. Rahman, N. C. Cheung and K. W. Lim
School of Electrical Engineering
University of New South Wales
Sydney 2052, Australia

Abstract- The modelling studies of a non linear solenoid with a view predict its dynamic response accurately is reported. This work was carried out with a view to converting an on/off solenoid into a proportional actuator. It investigates the magnetic characteristics of a solenoid and proposes an efficient dynamic model for the device. The modelling method leads to some simplification of analysis and implementation of the controller for the proportional actuator in a digital signal processor. The model has been used to design a proportional solenoid valve, the control and implementation of which is described in the this paper.

1. INTRODUCTION

Solenoid operated on-off actuators are used in a vast array of applications. Their popularity stems from the fact that they are simple and rugged in construction,, and relatively cheap to produce. These are normally used in the form of electrical contactors in relays, in the form of throttling devices in fluid/gas valves, and linear/rotary motional devices which operate in one of two positions - open and close. On the other hand, conventional proportional actuators based on the dc, stepper and other servo motors are high precision motion devices usually with a linear control characteristic. Such proportional actuators are more complex in construction, contain delicate moving and sensing elements, and is expensive to produce and maintain. Examples of applications include fluid flow controller in hydraulic servo systems, grasping motions in robot fingers, robot joints, positioning systems, machine tool drives, etc. If the simple, one coil solenoid could be converted into a propotional acuator with adequate accuracy, it may provide for a boon to the motion control industry.

The paper describes part of a project which aims to convert an on/off switching solenoid into a proportional actuator by the use of intelligent control [1,2]. The project is motivated by three considerations. Firstly, solenoids have a small and compact size, and the production cost is much lower than traditional proportional actuators. Secondly, solenoids have simple constructions, they are robust and maintenance free. Therefore higher reliability can be obtained. Finally, solenoids can easily incorporated into systems design due to its compact size and robust construction. However, to convert a switching solenoid into a proportional actuator is not a straight forward task because of the highly nonlinear magnetic characteristic of the device. It is also a variable reluctance device with its force derived in a complicated way from the change in its magnetic circuit. Though there has been a renewed interest in variable reluctance motors [3,4,5,6], there is little literature which deals with the modelling and precise control of solenoids.

The control characteristics of the solenoid is investigated first and an efficient model for the solenoid is then proposed. The model includes the non linear magnetic characteristics. The model has been subsequently used effectively in the analysis and implementation of a proportional solenoid [2]. It has been used to predict the electrical and mechanical dynamic response of a solenoid reasonably accurately

2. MAGNETIC CHARACTERISTICS OF THE LINEAR TRAVEL SOLENOID

The construction of a typical, limited travel solenoid is shown in fig. 1. The solenoid's plunger retracts inward when the coil is energised, and extends outward by releasing the stored energy from the spring. Total travel of the plunger is very short; in most cases, it is limited to about one centimetre. In order to achieve a high force to size ratio, most solenoids operate well into the non linear magnetic saturation region.

The flux-current-position relationships form basis of the characteristic motion of a solenoid. Since the solenoid operates well into the magnetic saturation region, its characteristics cannot be represented by simple linear equations. Figure 2 is a typical flux-current plot of a solenoid, with displacement x as a parameter. The curves show that flux linkage versus current is initially a linear relation. As the flux density increases, the curves become quite non linear. Also, different air gap positions have different degrees of non linearity.

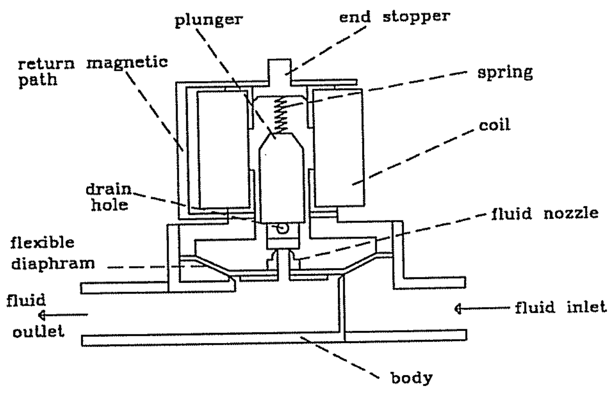


Fig. 1. A two stage solenoid valve.

3. THE SOLENOID MODEL

The solenoid is operated normally by applying a voltage to its coil, the voltage equation being expressed as:

$$V = Ri + \frac{d\lambda}{dt} \qquad (1)$$

where V is the terminal voltage and R is the resistance of coil. The flux linkage λ, is dependent on the coil current i, and the air gap distance x. Therefore the voltage equation can be rewritten as:

$$V = Ri + (L_e + \frac{\partial\lambda(x,i)}{\partial i})\cdot\frac{di}{dt} + \frac{\partial\lambda(x,i)}{\partial x}\cdot\frac{dx}{dt} \qquad [2]$$

where L_e is the inductance of the external circuit. Of the three terms in the equation (2), the first term is the resistive voltage drop, the second term is the inductive voltage due to change of current. The third term is known as a motional e.m.f. which is caused by the motion of the plunger. Equation (2) can only be solved if the magnetic characteristics of the solenoid are known.

The motion of the solenoid's plunger can be represented by a mass spring system:

$$m_p\ddot{x} = F_{mag} - K_S x - m_p g \qquad (3)$$

where m_p is the mass of the plunger, K_s is the spring constant, g is the gravitational constant, and F_{mag} is the force produced by magnetic field when the coil is energised. F_{mag} can be calculated from the co-energy W' of the magnetic circuit. The co-energy can be obtained from the integration of flux linkage λ against current I:

$$F_{mag} = \frac{\partial W'(x,i)}{\partial x} \qquad (4)$$

$$W'(x,i) = \int_0^i \lambda(x,i)\cdot di \qquad (5)$$

Since the variables i and x are fully independent and separable in relation to $\lambda(x,i)$, it is permissible to differentiate under the integral sign. Equation 4 and 5 become:

$$F_{mag} = \int_0^i \left.\frac{\partial\lambda(x,i)}{\partial x}\right|_{i=const} \cdot di \qquad (6)$$

For the instantaneous value of F_{mag}, when x does not change during a short period of time, equation (6) can be written as:

$$F_{mag} = \frac{\partial\lambda(x,i)}{\partial x}\cdot i \qquad (7)$$

From equations (2), (3), and (7) we can write the following set of nonlinear state equations (8 - 10) for the solenoid

$$\frac{dx}{dt} = v \qquad (8)$$

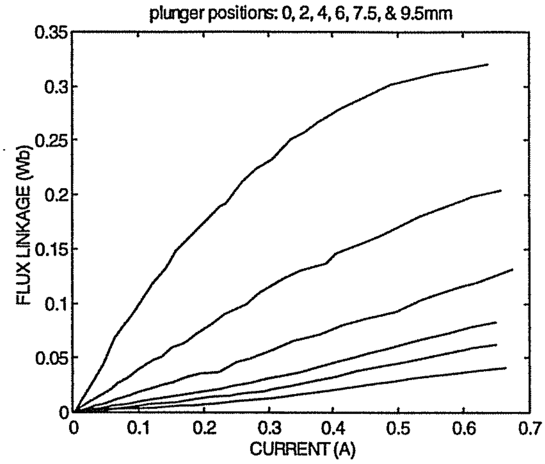


Fig. 2 Measurements of flux linkage versus current at different positions.

$$\frac{dv}{dt} = (\frac{\partial\lambda(x,i)}{\partial x}\cdot i - K_S x - m_p g)\cdot\frac{1}{m_p} \qquad (9)$$

$$\frac{di}{dt} = (V - Ri - \frac{\partial\lambda(x,i)}{\partial x}\cdot\frac{dx}{dt})\cdot\frac{1}{L_e + \frac{\partial\lambda(x,i)}{\partial i}} \qquad (10)$$

In equations (8) to (10), $\partial\lambda/\partial x$ and $\partial\lambda/\partial i$ are obtained from a model of the magnetic characteristics. A parsimonious model of these characteristics is the key to the usefulness of these equations.

Several curve fitting techniques are available for describing the magnetic characteristics of a variable reluctance machine [7]. However, they are either too complicated to implement in real time [8,9,10], or not sufficiently accurate for proportional solenoids. The curve fitting technique in [11] provides a balance between modelling accuracy and computation efficiency for a switched reluctance motor, but a different modelling description is required for the solenoid. All these methods are designed for use in variable reluctance motors only.

In this paper, we propose to model the magnetic characteristics of a solenoid by dividing its operation into linear and saturated regions. A simple relation based on linear magnetic principles is used in the linear model. This is described first. For the saturated region, a parabolic curve approximation is used for the flux linkage versus current curves; and an inverse function approximation is used for the flux linkage versus position curves. This is described next. The proposed model provides a good balance between accuracy and computational efficiency.

3.1 THE SOLENOID IN THE LINEAR REGION

A description of the linear region can be obtained by using standard electromagnetic principles based on linear magnetic circuit [10]. Consider a solenoid which operates within its linear region, and has the following parameters:

Equivalent cross sectional area for iron core and air gap: A
Effective core length: l Air gap length: x
No. of turns of coil: N Current through coil: i
Permeability of freespace: μ_o Relative permeability: μ_r
Magnetic flux: Φ

$$Ni = \frac{l\Phi}{\mu_0 \mu_r A} + \frac{x\Phi}{\mu_0 A} \qquad (11)$$

The solenoid can be viewed as a variable reluctance magnetic circuit, with a m.m.f. equation as in equation (11). It contains two terms; the first term describes the m.m.f. drop due to the iron core, and the second term is the m.m.f. drop due to the air gap. μ_r is treated as constant, since it operates below the saturation region. By substituting $\lambda = N\Phi$, equation (10) can be rearranged as:

$$\lambda(x,i) = \frac{\mu_0 A N^2 i}{\frac{l}{\mu_r} + x} \qquad (12)$$

By introducing two constant terms, K_a and K_b, equation (12) can be simplified as:

$$\lambda(x,i) = \frac{K_a}{K_b + x} \cdot i \qquad (13)$$

K_b is roughly constant, in spite of the fact that it changes with the plunger position. This is because K_b is much smaller than x for most of the time, and has little effect on the overall equation, except when x is close to zero. When x is very close to zero, the percentage of variation of K_b is very small and may be neglected.

Since leakage flux exists in solenoid, and a proportional amount of current is being used up to produce this leakage flux, (13) has to be compensated by *offset1* as shown by (14):

$$\lambda(x,i) = \left(\frac{K_a}{K_b + x} - \text{offset1}\right) \cdot i \qquad (14)$$

K_a, K_b, and *offset1* determines the linear portion of magnetic characteristics; these parameters are obtained by curve fitting (14) to the measured values of flux linkage versus position at $i = i_s$, by using a standard mathematical software package.

3.2 FLUX LINKAGE VERSUS CURRENT

Equation (14) is the general equation for describing the magnetic behaviour of a variable reluctance solenoid in the linear region. For the non linear region, the non linear plots are approximated by parabolic curves as shown in fig 3. The parabolic curve has a curvature of K_p with an origin point of $O(i_o, \lambda_o)$. It is represented by equation (15).

$$(\lambda_p - \lambda_0)^2 = 4K_p(i_p - i_0) \qquad (15)$$

In fig. 3, λ_s is the flux saturation boundary, above which saturation will occur, $P(i_p, \lambda_p)$ is a point along the flux-current curve with position equal to x. L_x is the gradient of the linear part of the flux-current plot, and is obtained by:

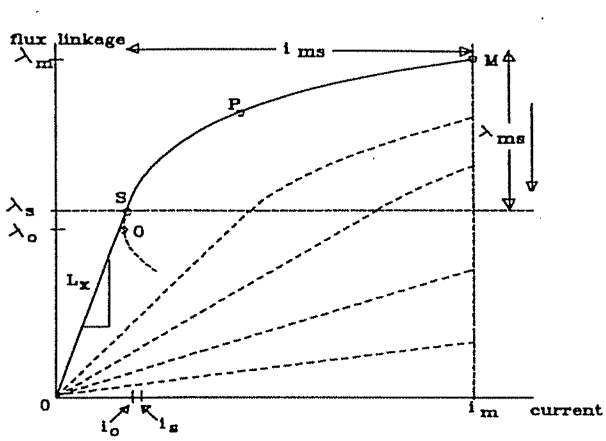


Figure 3. Curve fitting the flux-current characteristic of a solenoid by linear and parabolic approximation.

$$L_x = \frac{\lambda}{i} = \frac{K_a}{K_b + x} - \text{offset1} \qquad (16)$$

The saturation current i_s, can be obtained from the gradient L_x by:

$$i_s = \frac{\lambda_s}{L_x} \qquad (17)$$

In order to have a smooth transition between the linear and the non linear region, the gradient of the parabolic curve at point $S(i_s, \lambda_s)$ must be equal to the gradient in the linear region. Using this rule, i_o and λ_o can be found:

$$i_0 = i_s - \frac{K_p}{L_x^2} \qquad (18)$$

$$\lambda_0 = \lambda_s - \frac{2K_p}{L_x} \qquad (19)$$

The curvature of the curve is set so that the locus of the parabola will intersect point $M(i_m, \lambda_m)$:

$$K_p = \frac{\lambda_{ms}^2}{4\left(i_{ms} - \frac{\lambda_{ms}}{\lambda_m}\right)} \qquad (20)$$

where $\lambda_{ms} = \lambda_m - \lambda_s$
$i_{ms} = i_m - i_s$

This method constructs a parabolic curve based on two coordinates, $M(i_m, \lambda_m)$ and $S(i_s, \lambda_s)$ only. Since λ_s and i_m are predetermined values, therefore only i_s and λ_m needs to be determined. i_s can be obtained from (15). whereas λ_m can be obtained from a simple look up table of flux against position at i_m. Alternatively, λ_m can be calculated from the following equation:

$$\lambda_m = \frac{K_c}{K_d} - offset2 \quad (21)$$

K_c, K_d, and *offset2* are found by curve fitting (21) to the flux linkage against position curve when $i=i_m$.

With this approach, flux linkage against current relation can be calculated by the following parameters with (16)-(21): K_a, K_b, K_c, K_d, λ_s, i_m, *offset1*, and *offset2*.

3.3 FLUX LINKAGE VERSUS POSITION

Figure 4 is a typical curve of flux linkage against position for a solenoid. The curve in the linear region can be represented by equation (13). For the saturated region, the curve can be approximated by an inverse function. The function has the following form:

$$\lambda_p = \frac{K_1}{K_2 + x_p} - K_3 \quad (22)$$

To have the above function pass through point $M(0,\lambda_m)$, K_1, K_2, and K_3 are specified as follows:

$$K_1 = K \cdot S_c \cdot \lambda_m \,;\, K_2 = K \,;\; K_3 = (S_c - 1)\cdot\lambda_m \quad (23)$$

The value of K and S_c are selected so that the curve of described by equation (22) passes through point S and S_δ. By using the equation in the linear region, x_s can be obtained:

$$x_s = \frac{K_a}{offset1 + \frac{\lambda_s}{i}} - K_b \quad (24)$$

Point $Sd_\delta(x_{s\delta},, \lambda_{s\delta}$)lies to the left and very close to point $S(x_s, \lambda_s)$. It is defined by the following relations:

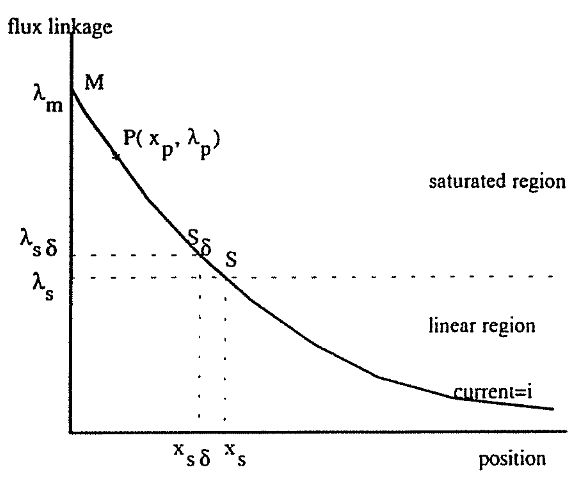


Figure 4. Curve fitting the flux-position characteristic of a solenoid by inverse functions approximation.

$$x_{s\delta} = x_s - \delta \quad (25)$$

$$\lambda_{s\delta} = (\frac{K_a}{K_b + x_{s\delta}} - offset1)\cdot i \quad (26)$$

By referring to (22) and (23), the following equations can be formed:

$$\lambda_s = \frac{K \cdot S_c \cdot \lambda_m}{K + x_s} - (S_s - 1)\lambda_m \quad (27)$$

$$\lambda_{s\delta} = \frac{K \cdot S_c \cdot \lambda_m}{K + x_{s\delta}} - (S_c - 1)\lambda_m \quad (28)$$

Once λ_m is obtained, the above two equations are used to solve K and S_c. K and S_c can be calculated as follows:

$$S_c = \frac{ac - c}{\frac{ac}{b} - 1} \qquad K = x_s \cdot \frac{S - b}{b}$$

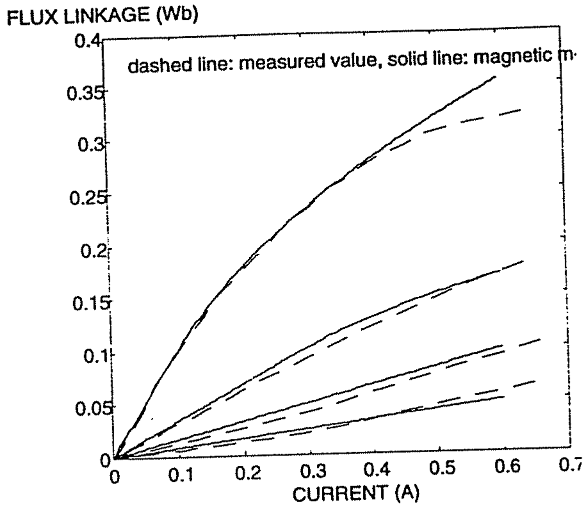
$$\text{where} \quad a = \frac{x_s}{x_{s\delta}};\; b = 1 - \frac{\lambda_s}{\lambda_m};\; c = 1 - \frac{\lambda_{s\delta}}{\lambda_m} \quad (29)$$

This method constructs a *1/x* function curve based on points *M, S* and S_δ only. λ_m can be obtained from a table of flux linkage against current when position $x=0$. Alternatively, λ_m can be calculated from the parabolic curve as described in the previous section. In this case, K_p, λ_o, and i_o at $x=0$ should be predetermined to enable faster calculation. With this approach, flux linkage versus position can be calculated by K_a, K_b, *offset1*, and λ_s by using (22)-(26) and (29).

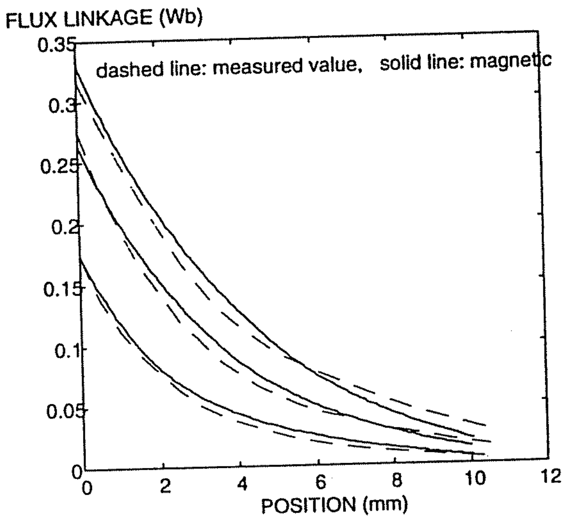
The proposed modelling method exploits the fact that the magnetic circuit of a solenoid contains two operating regions: linear and saturated. For each operating region, the magnetic characteristic is described by a lowest order approximation function just sufficient for the required accuracy. In this way a model with the least complex describing functions is constructed. This model is computationally efficient, and is therefore very suitable for real-time computation of the solenoid's magnetic characteristics. For the same accuracy, there are other higher order and more general approximation methods available (e.g. third order bi-cubic spline, sine-cosine description), but they require more complex calculations, and are less suitable for real time applications [9,10,11].

4. MEASUREMENT OF MAGNETIC CHARACTERISTICS

The flux Vs position Vs current characteristic of an industrial solenoid were measured by using ac excitation. The coil was excited by sinusoidal voltage for each position and the hysteresis loops traversed by the magnetic circuit were computed. By knowing the number of turns of the solenoid and the by joining the vertrices of the hysteresis loops, the charactersistics were obtained. For the solenoid chosen, a sample of the characteristics are indicated by the solid curves of fig. 5. The solenoid data are indicated in Table I.



(a)



(b)

Figure 5. Comparison of the magnetic model with the actual measured values: (a) flux linkage vs. current, at x=0, 2.5, 5, & 7.5mm, (b) flux linkage vs. position, at i=0.2, 0.4, & 0.6A.

TABLE I

Make	Goyen Controls
Type	Two stage switching solenoid valve
Stroke length	10mm
Operating voltage	24V d.c.
Maximum current	0.6A
Resistance	40ohms
Inductance	0.35-1.1H
No of turns of coil	2240

5. MODELLING RESULTS

The six parameters mentioned in the earlier section were obtained curve fitting (14)-(21) to the measured data. The modelled flux linkage Vs position Vs current characteristics are indicated along the experimental data by the broken curves. Close correspondence between the measured and the modelled results are easily seen.

Computation of the force on the solenoid plunger is found from the co-energy of (7). The computed and the measured force data are shown in figure 6. Again, the predicted force is found to be reasonably accurate, except at the extreme positions of the plunger. Such extreme positions may be avoided in actual operation without significant loss of operational range.

Once the force on the plunger is determined, the dynamics of the solenoid is found by solving (8)-(10). Figures 7(a) and 7(b) show the modelled and experimental data on the position and the current dynamics of the solenoid when its supply voltage is turned on suddenly.

6. DEVELOPMENT OF A PROPORTIONAL SOLENOID

The representation of the solenoid characteristics in the form proposed in the paper allowed a proportional controller to be developed that treated the solenoid characteristic data in the same way as in the modelling. The modelling is thus a suitable vehicle to indicate whether a controller will be successful. A control system as indicated in the fig. 8 was developed using a digital signal processor for the proportional controller. It relates the measured position error to a force command which in turn relates to a current reference for the solenoid. These relationships are defined by (14)-(21). Figure 9(a) show the plunger's ability to follow a triangular (moving) position reference in simulation. Figure 9(b) show the corresponding data from actual measurements when the solenoid is controlled as indicated in figure 8. The position following performance of the solenoid is satisfactory both in modelling and in implementation.

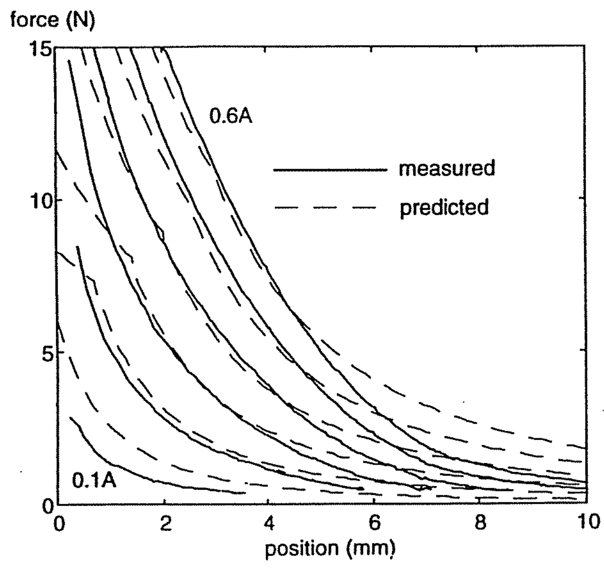


Figure 6. Comparison of predicted force with actual measurement.

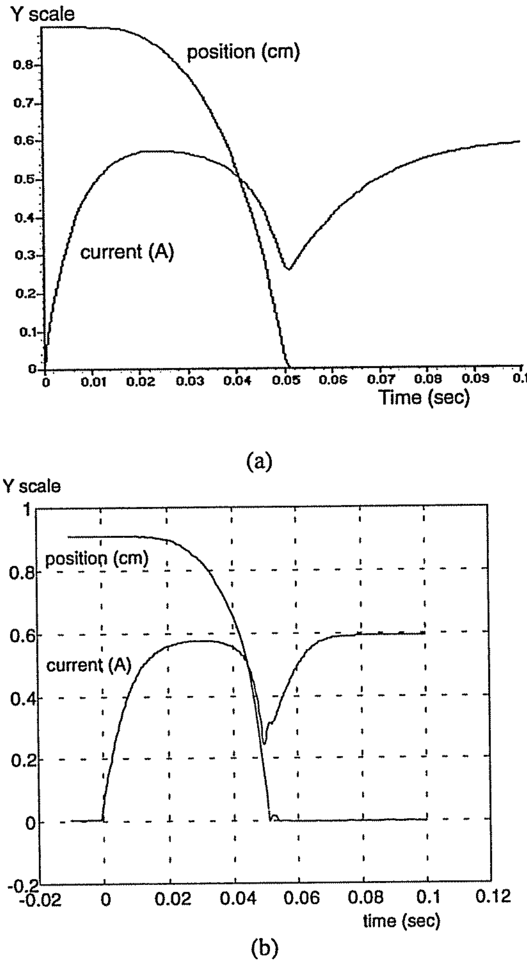


Figure 7. Dynamic response of the solenoid: (a) simulation result, and (b) actual measurement

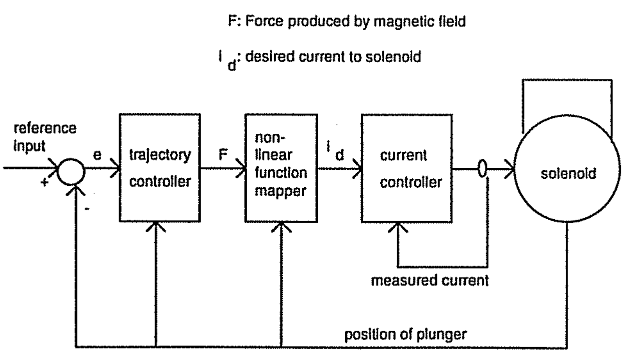


Figure 8. Block diagram of the control system.

7. CONCLUSION

This paper has described a model of a highly non linear solenoid and has predicted its dynamic response reasonably accurately. The model uses the motor characteristic data and provide a set of parameters which are subsequently used by a controller which drives the solenoid as a proportional actuator. The controller works well and the dynamic response of the overall control system is predicted accurately.

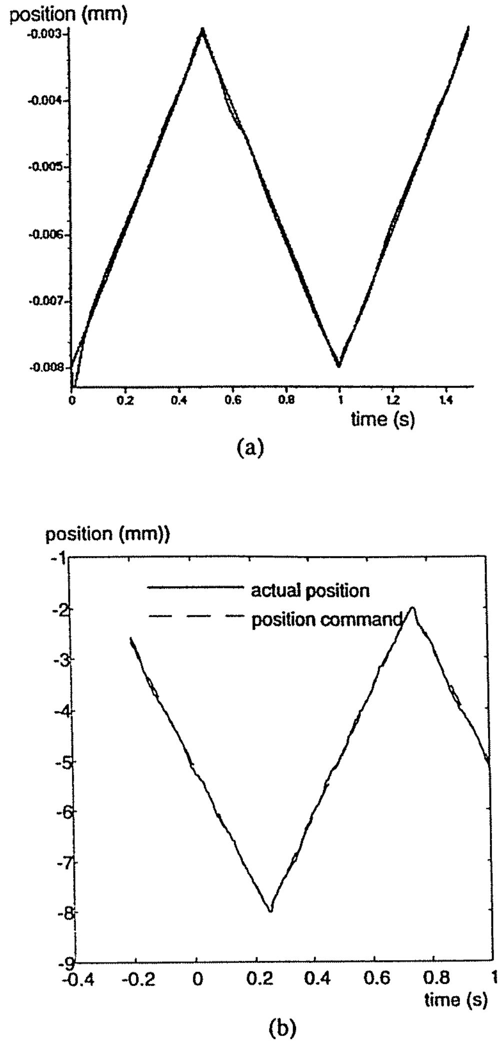


Figure 9. Simulation and actual results of position following. (a) simulated ramp position following, (b) experimental.

REFERENCES

[1] K.W. Lim, N.C. Cheung, M.F. Rahman, "Proportional control of a solenoid actuator," IEEE Industrial Electronics Society annual general meeting, Sept 1994, Bologna, Italy.

[2] M.F. Rahman, N.C. Cheung, K.W. Lim, "Converting a switching solenoid into a proportional actuator," International Power Electronics Conference, IPEC'95, Yokohama, Japan.

[3] F. Filicori, C.G.L. Bianco, A. Tonielli, "Modelling and control strategies for a variable reluctance direct drive motor," IEEE Trans. on Industrial Electronics, vol. 40,

no. 1, pp105-115, February, 1993.

[4] G.S. Buja, and M.A. Valla, "Control characteristics of the SRM motor drives - Part I, operation in the linear region," IEEE Trans. on Industrial Electronics, vol. 38, no. 5, pp 313-321, October, 1991.

[5] G.S. Buja, and M.A. Valla, "Control characteristics of the SRM motor drives - Part II, operation in the saturated region," IEEE Trans. on Industrial Electronics, vol. 41, no. 3, pp 316-325, June, 1994.

[6] A. A. Goldenberg, I. Laniado, P. Kuzan and C. Zhou, "Control of Switched Reluctance Motor Torque For Force Control Applications", IEEE Transactions on Industrial Electronics, Vol. 41, No. 4, pp 461-466, Aug 1994.

[7] H.D. Chai, "A mathematical model for single stack step motors", IEEE Trans Power Apparatus ans Systems, PAS-94, pp1508-1517, 1975.

[8] D.A. Torrey and J.H. Lang, "Modelling a non linear variable reluctance motor drive," IEE Proc. vol. 137, pt. B, no. 5, pp 314-326, Sept 1990.

[9] D.G. Manzer, M. Varghese, and J.S. Throp, "Variable reluctance motor characterization," IEEE Trans. on Industrial Electronics, vol. 36, no. 1, pp 56-63, Feb 1989.

[10] J.M. Stephenson and J. Corda, "Computation of torque and current in doubly salient reluctance motors from non linear magnetisation data", IEE Proc., pt. B, vol. 126, pp 393-396, 1979.

[11] T.J.E. Miller and M. McGilp, "Non linear theory of the switched reluctance motor for rapid computer aided design," IEE Proc. vol 137, pt. B, no. 6, pp 337-346, Nov 1990.