PERFORMANCE COMPARISON AND EVALUATION OF DIGITAL DISTANCE PROTECTION ALGORITHMS

K. K. Li* C. Cheung* Y. Q. Xia**

*Hong Kong Polytechnic, Kowloon, HONG KONG.
**Tianjin University, Tianjin, China.

Abstract

This paper outlines the theory of three modern digital distance relay algorithms: the modified finite transform algorithm, the time domain algorithm and the kalman filtering algorithm. The performance of these algorithms are evaluated and compared by simulation in high level language. The result shows that the modified finite transform algorithm is most suitable for digital relay application. The amount of computations is acceptable and is suitable for pure digital application.

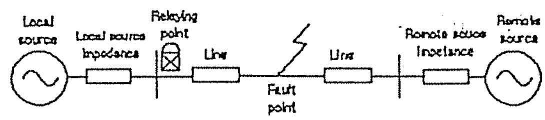
1. Introduction

During fault conditions, the voltage and current waveforms are usually distorted and consist of numerous harmonics and reflected travelling wave components. However, they contain the basic information about the fault. A high speed distance relay is therefore required to estimate the fundamental frequency components from the corrupted voltage and current signals following the fault occurrence. These components are then used to determine the apparent impedance to the fault, and hence the fault location. This process should be performed as quickly as possible.

Many digital relay algorithms have been developed. They all aim at ultra-high operating speed, efficient, and high stability. It is thus necessary to evaluate their performance under various conditions. Three algorithms are being studied. They are :

(i) Finite Transform Algorithm [1, 2, 6],
(ii) Time Domain Algorithm [3], and
(iii) Kalman Filtering Algorithm [4, 5].

The study case is based on a 400 KV line of length 128 Km interconnecting two power sources as shown in Fig. 1. Fault transient voltage and current waveforms at the relaying point are generated by software using a method similar to that developed by A.T. Johns [7].



MVA base = 600 MVA
Fault level at relaying end = 35 GVA
Fault level at remote end = 35 GVA
Pre-fault relaying end busbar voltage = $1.0 \angle 0^\circ$
Pre-fault remote end busbar voltage= $1.0 \angle 0^\circ$
Total line impedance = $36.8 \angle 86^\circ$ Ω

Figure 1 System diagram for study case

2. Finite Transform Algorithm [1, 2, 6]

In this algorithm the information of the line is obtained by suitably processing a small window of information. The objective of these algorithm is to extract the components of line impedance referring to the fundamental frequency irrespective of the distortions in voltage and current waveforms.

In a simplified transmission line circuit, the voltage and current at the relaying point can be related by eqn. 1 irrespective of the waveforms.

$$v_r = Z(p)\, i_r \qquad (1)$$

The voltage v_r can be considered as the summation of an arbitrary number of components v_{r1} , v_{r2} , ..., such that $v_r = v_{r1} + v_{r2} + ...$ Superposition can be used to relate the total current i_r to the components of currents which would flow in the circuit in response to the separate application of each of the voltage components. That is, if the current flow in response to v_{r1} , v_{r2} , ... are i_{r1} , i_{r2} , ..., respectively, the total current is given by $i_r = i_{r1} + i_{r2} + ...$ Each pair of components is related in the same form as eqn. 1.

The effect of conductance and capacitance may be neglected for short transmission lines. Eqn. 1 may be rearranged as shown below

$$V_r(t) = R\,i_r(t) + L\,di_r(t)\,/\,dt \qquad (2)$$

Solving eqn. 2 by finite transform [5] gives

$$\bar{v}_r(j\omega_e,t) = R\,\bar{i}_r(j\omega_e,t) + j\omega_e L\,\bar{i}_r(j\omega_e,t) + L\,\{i_r(t)\exp(-j\omega_e t) - i_r(t-T_w)\exp[-j\omega_e(t-T_w)]\} \qquad (3)$$

Decomposing the finite transform of voltage and current into real and imaginary parts gives,

$$\begin{aligned} \bar{v}_r(j\omega_e,t) &= \bar{v}_{r1} - j\bar{v}_{r2} \\ \bar{i}_r(j\omega_e,t) &= \bar{i}_{r1} - j\bar{i}_{r2} \end{aligned} \qquad (4)$$

Substituting eqn. 4 into eqn. 3, R and L can then be expressed as

$$\begin{bmatrix} R \\ L \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} S_i & -C_o \\ -\bar{i}_{r2} & \bar{i}_{r1} \end{bmatrix}\begin{bmatrix} \bar{v}_{r1} \\ \bar{v}_{r2} \end{bmatrix} \qquad (5)$$

where $\Delta = \bar{i}_{r1} S_i - \bar{i}_{r2} C_o$.

$$\begin{aligned} C_o &= i_r(t)\cos(\omega_e t) - i_r(t-T_w)\cos[\omega_e(t-T_w)] + \omega_e \bar{i}_{r2} \\ S_i &= i_r(t)\sin(\omega_e t) - i_r(t-T_w)\sin[\omega_e(t-T_w)] - \omega_e \bar{i}_{r1} \end{aligned} \qquad (6)$$

Hence a measurement of the line impedance as seen from the relaying point can be obtained.

The impedance calculated using this algorithm for A phase to earth (A-E) faults at various locations of the line are shown in Fig. 2 - 7. Computations are based on a sampling frequency of 4 kHz, and a sampling window T_w of 16 samples. Only a low pass filter with cut off frequency of 2 kHz is used to avoid aliasing error.

From these diagrams, it is found that the computed impedance converged in about 4 to 7 ms. This demonstrates the algorithm's strong ability to extract the fundamental components even with a very distorted input.

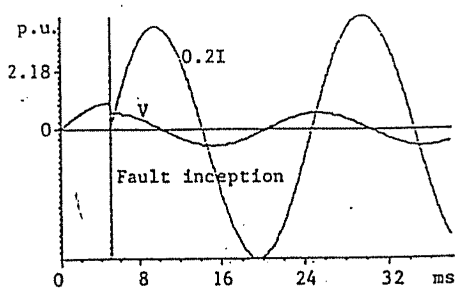


Fig. 2 Voltage & Current Waveforms
A-E Fault at 12.8 Km

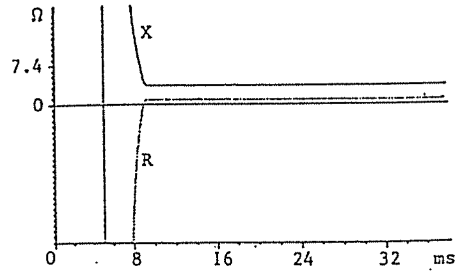


Fig. 3 Impedance Measurement
A-E Fault at 12.8 Km

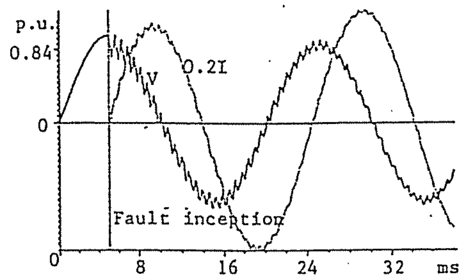


Fig. 4 Voltage & Current Waveforms
A-E Fault at 64 Km

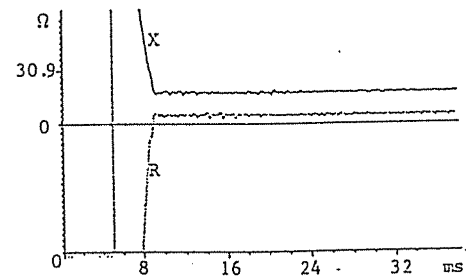


Fig. 5 Impedance Measurement
A-E Fault at 64 Km

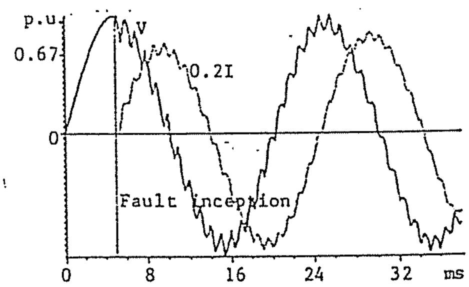


Fig. 6 Voltage & Current Waveforms
A-E Fault at 115 Km

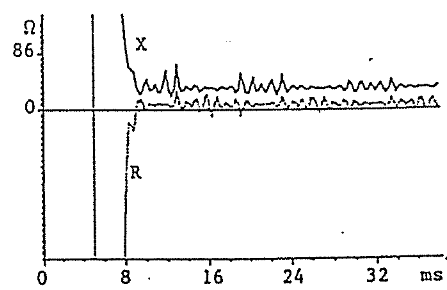


Fig. 7 Impedance Measurement
A-E Fault at 115 Km

3. Time Domain Algorithm [3]

The Time Domain Algorithm is based on approximate voltage and current polynomial expansions with distance, which is derived by integrating in the time domain the partial differential equations for a distributed line. For a distributed constant line of overall length 'l', in which the inductance per unit length of the line is represented by L, the capacitance by C, the resistance by R, and the conductance by G as shown in Fig. 2, the voltage $e(x, t)$ and the current $i(x, t)$ at position x and time t satisfy the partial differential eqns. 7 :

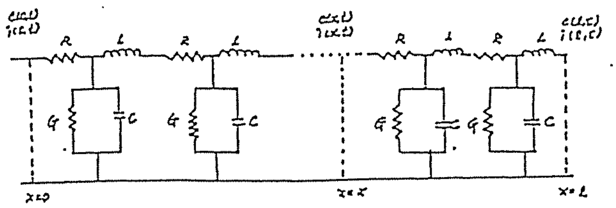


Fig. 8 Distributed Constant Line

$$-\frac{\partial}{\partial x}e(x,t) = Ri(x,t) + L\frac{\partial}{\partial t}i(x,t)$$
$$-\frac{\partial}{\partial x}i(x,t) = Ge(x,t) + C\frac{\partial}{\partial t}e(x,t) \qquad (7)$$

For a distance relay, at the location of fault,

$$e(x,t) = 0$$
$$\frac{\partial}{\partial t}e(x,t) = 0 \qquad (8)$$

Eqn. 7 can be integrated in time domain by using the operator method by assuming the initial condition to be zero. Eqns. 9 and 10 can then be derived:

$$e(x,t) = e(0,t) - \{Ri(0,t) + L\frac{\partial}{\partial t}i(0,t)\}x + \{RGe(0,t) + (RC+LG)\frac{\partial}{\partial t}e(0,t) + LC\frac{\partial^2}{\partial t^2}e(0,t)\}\frac{x^2}{2!} - \ldots\ldots = 0 \qquad (9)$$

$$0 = \frac{\partial}{\partial t}e(0,t) - \{R\frac{\partial}{\partial t}i(0,t) + L\frac{\partial^2}{\partial t^2}i(0,t)\}x + \{RG\frac{\partial}{\partial t}e(0,t) + (RC+LG)\frac{\partial^2}{\partial t^2}e(0,t) + LC\frac{\partial^3}{\partial t^3}e(0,t)\}\frac{x^2}{2!} - \ldots\ldots \qquad (10)$$

The number of terms of the polynomials to be retained is determined by considering filter characteristics and line parameters such as capacity and length. The coefficients of the polynomials are the function of voltage, current and their derivatives, which can be efficiently computed by curve fitting and analytical differentiation. For short overhead line, G and C may be neglected. Therefore,

$$e(t) - \{Ri(t) + L\frac{d}{dt}i(t)\}x = 0 \qquad (11)$$

$$\frac{d}{dt}e(t) - \{R\frac{d}{dt}i(t) + L\frac{d^2}{dt^2}i(t)\}x = 0$$

$$x = \frac{e(t)}{Ri(t) + L\frac{d}{dt}i(t)} \qquad (12)$$

The fault distance is computed based on eqn. 15 for A phase to earth faults at various locations are plotted in Fig. 9 to 14.

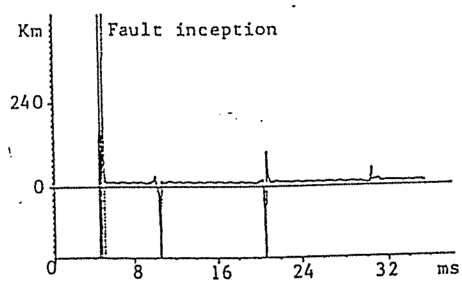


Fig. 9 A-E Fault at 12.8 Km

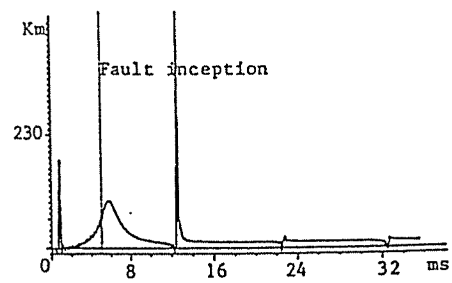


Fig. 10 A-E Fault at 12.8 Km
(With 100 Hz Low Pass Filter)

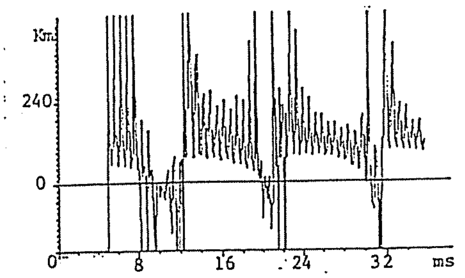


Fig. 11 A-E Fault at 64 Km (without filter)

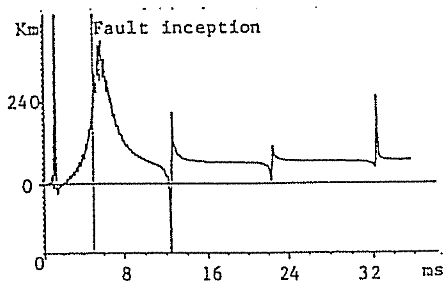


Fig. 12 A-E Fault at 64 Km
(With 100 Hz Low Pass Filter)

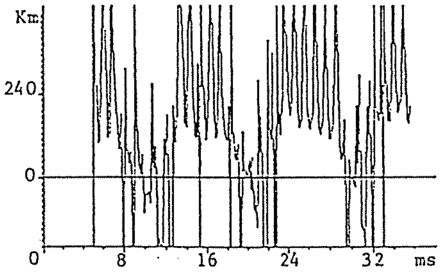


Fig. 13 A-E Fault at 115 Km (without filter)

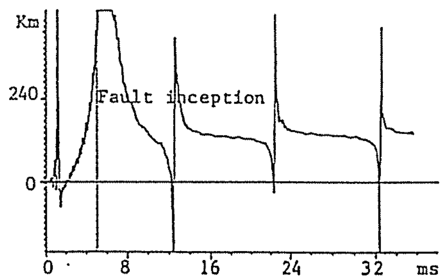


Fig. 14 A-E Fault at 115 Km
(With 100 Hz Low Pass Filter)

From the above results, it can be observed that this algorithm is very sensitive to harmonics and high frequency components. The measurement for a distant fault is inaccurate due to the presence of large high frequency disturbances. The fault distance x calculated is dependent on the value of *e(t)*. When *e(t)* is very small or zero, the error becomes very large. This repeats for approximately every 10 ms when *e(t)* changes polarity. This algorithm cannot be applied unless a low pass filter is used to cut off all higher harmonics and high frequency components.

4. Kalman Filtering Algorithm [4]

4.1 Theory of Kalman Filters

Kalman filters, as recursive optimal estimators, are used to optimally estimate the fundamental frequency voltage and current components. The mathematical model of the Kalman filter is based on the state space concept. It is formed based on the knowledge of the statistics of initial conditions, the process noise model, and the measurement noise model. The signal $s(t)$ is described by two state variables, x_1, the in-phase component, and x_2, the quadrature component as shown in eqn. 13.

$$s(t) = [\cos(\omega_o t), \ -\sin(\omega_o t)] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (13)$$

The noisy input is processed recursively. The filter is initialized with an initial estimate of the signal and its error covariance. The data obtained is used to update or refine the filter's previous estimate, until, a steady-state condition is reached.

The mathematical model of the signals to be estimated is assumed in the form of eqn. 14

$$x_{k+1} = \phi_k x_k + w_k \quad (14)$$

The measurement process Z_k is in the form

$$z_k = H_k x_k + v_k \quad (15)$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k \hat{x}_k^-) \quad (16)$$

where ϕ_k, H_k *are the system and measurement equations respectively.*

The calculation process is recursive in nature and is initiated by a priori estimate $\hat{x}_k^-$ and its error covariance. The Kalman filter gain K_k depends on the error covariance, system and measurement equations, and system and measurement noise models. However, its computation is independent of measurement, and can be computed off-line.

4.2 Voltage and Current Models

The voltage and current models used are shown below

4.2.1 Voltage models - both faulted and unfaulted phases.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k \qquad (17)$$

$$z_k = [\cos(\omega_o k\Delta t),\ -\sin(\omega_o k\Delta t)] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + v_k \qquad (18)$$

4.2.2 Current Models - faulted phase

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\beta\Delta t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_k + \begin{bmatrix} 0 \\ 0 \\ w_k \end{bmatrix} \qquad (19)$$

$$z_k = [\cos(\omega_o k\Delta t),\ -\sin(\omega_o k\Delta t),\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_k + v_k \qquad (20)$$

4.2.3 Current models - unfaulted phase

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k \qquad (21)$$

$$z_k = [\cos(\omega_o k\Delta t),\ -\sin(\omega_o k\Delta t)] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + v_k \qquad (22)$$

where x_1, x_2, x_3 are the corresponding states of voltages and currents.

The states of voltages and currents for an A-E fault at 115 Km from the relaying point is shown in Fig. 15 - 20. In can be seen from these diagrams that the states of voltage and currents converged generally in less than 10 ms. The values of states can also be used to identify the faulted phase(s).

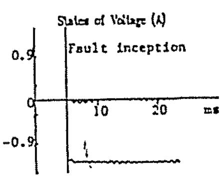


Fig. 15

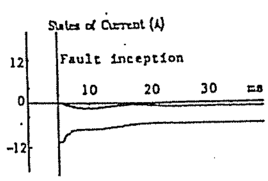


Fig. 16

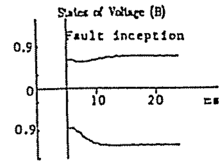


Fig. 17

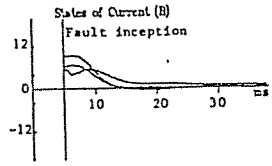


Fig. 18

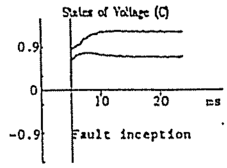


Fig. 19

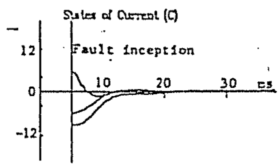


Fig. 20

4.3 <u>Distance Relaying by Kalman Filtering Technique</u>

At the relaying point,

$$\begin{aligned} V &= V_1 + jV_2 \\ I &= I_{s1} + jI_{s2} \end{aligned} \qquad (23)$$

The apparent impedance can be calculated as

$$\begin{aligned} Z_{app} &= R_{app} + jX_{app} \\ &= \frac{V_1 + jV_2}{I_{s1} + jI_{s2}} \end{aligned} \qquad (24)$$

The impedance calculated for A-E faults at various locations are shown in Fig. 21 to 23. For faults in 10% and 50% of line length, the impedance computed converged in less than 5 ms. However, this algorithm shows an increase in computing time (up to about 15 ms.) when the fault is far away from the relaying point (say, 90% of line length) due to the presence of exceptional severe high frequency components.

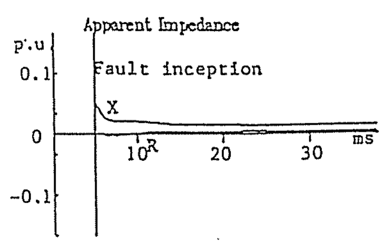


Fig. 21 A-E Fault at 12.8 Km

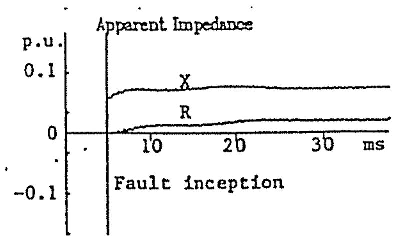


Fig. 22 A-E Fault at 64 Km

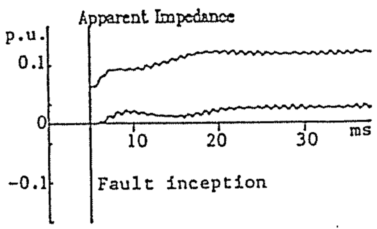


Fig. 23 A-E Fault at 115 Km

5. Conclusion

All the three algorithm mentioned in this paper have fast response time ranging from 4 to 15 ms. As can be seen in various diagrams, the speed is affected by the presence of noise and high frequency components. The Finite Transform algorithm, however, is found to be least affected by these factors and the calculated impedance converged within 4 to 7 ms.

In choosing a suitable algorithm for the digital relay, other factors, e.g. stability, computation, etc., has to be taken into consideration.

The Time Domain algorithm is not stable without the use of filters. The inclusion of conductance and capacitance and higher order terms in the differential equation can improve the stability, but this will also increase the amount of computation and hence the burden of processor thus making it unsuitable for real time application. This algorithm seems to be more suitable for post-fault analysis.

The Kalman Filtering algorithm has to be initialized with an initial estimate of the signal and its error covariance based on past statistics. These parameters is different in different networks and is therefor difficult to apply. One way to improve its performance is to design the filter to be able to adapt to various situation. This will also increase the amount of computation and the burden of the processors.

The modified Finite Transform algorithm is the most stable among the three algorithm. Techniques [8] can be used to reduce the amount of computation, and simulation [6] has proved that all the computations can be completed within one sampling interval (250 μs) without difficulties. This makes it suitable for pure digital application.

Acknowledgments

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